MATHEMATICS-IX

Module - 5

INDEX

Shrivastava Classes, D-27, Near JVTS Garden,Chattarpur Extension
New Delhi - 110074
9810934436 , 8076575278 , 8700391727

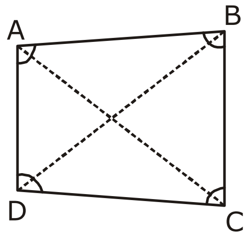
QUADRILATERALS AND PARALLELOGRAM

INTRODUCTION

In the previous chapter, we studied triangles and their properties. We know that a triangle is formed by joining any three non-collinear points in a plane. Similarly, if we join any four points in a plane, taken in order, no three of which are collinear then they form a quadrilateral. If we look around us, we will find a number of objects which are in the shape of a quadrilateral, for example- wall of a room, top of a book, blackboard, top of a table etc.

In the present chapter, we shall study quadrilaterals and their different kinds as parallelogram, rectangle, rhombus and square.

Quadrilateral. A plane figure formed by joining four points in an order, no three of which are collinear is called a quadrilateral. A quadrilateral has four vertices, four sides and four angles. It has two diagonals.



In the adjacent figure, ABCD is a quadrilateral. It has four vertices A, B, C and D. It has four sides AB, BC, CD and DA. It has four angles $\angle A$, $\angle B$, $\angle C$ and $\angle D$. It has two diagonals AC and BD.

A quadrilateral ABCD is also written as 'quad ABCD' or '$\square$ ABCD' in short.

Terms related to a Quadrilateral.

1. **Adjacent sides.** Any two sides of a quad. which have one common vertex are called adjacent sides or consecutive sides.

   In quad. ABCD following are the pairs of adjacent sides -

   AB, BC; BC, CD; CD, DA; DA, AB.

2. **Opposite sides.** Any two sides of a quad.which have no common vertex are called opposite sides.

   In quad. ABCD following are the pairs of opposite sides-

   AB and CD; BC and AD.

3. **Consecutive angles.** Any two angles of a quad. which have one common arm are called consecutive angles.

   In quad. ABCD, $\angle A$ and $\angle B$; $\angle B$ and $\angle C$; $\angle C$ and $\angle D$; $\angle D$ and $\angle A$ and consecutive angles.

4. **Opposite angles.** Any two angles of a quad. Which have no common arm are called opposite angles.

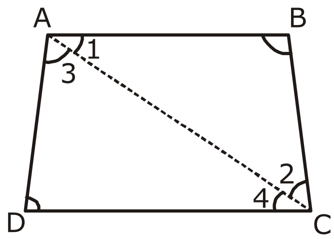
   In quad. ABCD, $\angle A$ and $\angle C$; $\angle B$ and $\angle D$ are pairs of opposite angles.

Angle sum property of a quadrilateral.

Theorem 1. The sum of all angles of a quadrilateral is 360º.

Given. A quad. ABCD.

To prove. $\angle A + \angle B + \angle C + \angle D = 360^\circ$.



Construction. Join AC.

Proof. In ΔABC,

$\angle 1 + \angle B + \angle 2 = 180^o$ (angle sum property of a Δ) .....(1)

In ΔADC,

$\angle 3 + \angle D + \angle 4 = 180^o$ (Angle sum property of a Δ) .....(2)

Adding equations (1) and (2), we get

$\angle 1 + \angle B + \angle 2 + \angle 3 + \angle D + \angle 4 = 180^o + 180^o$

$\Rightarrow \quad (\angle 1 + \angle 3) + \angle B + (\angle 2 + \angle 4) + \angle D = 360^o$

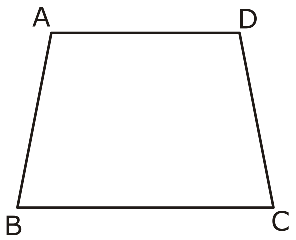
$\Rightarrow \quad \angle A + \angle B + \angle C + \angle D = 360^o$

Types of Quadrilaterals.

There are different types of quadrilaterals which are discussed below -

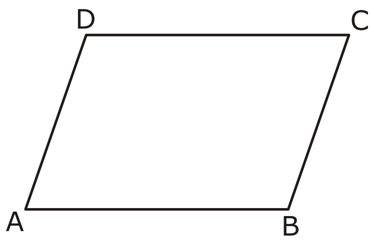
1. **Trapezium.** A quad. in which one pair of opposite sides is parallel is called a trapezium. In the adjacent figure, ABCD is a trapezium with AD||BC. In short we write it as 'trap. ABCD'.

A trap. is said to be isosceles trap. if its non-parallel sides are equal.



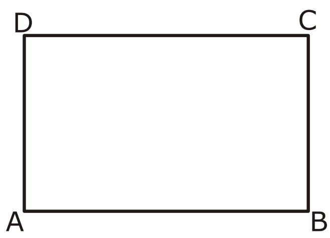
Thus trap. ABCD is isosceles if AB = DC.

2. **Parallelogram.** A quad. in which both the pairs of opposite sides are parallel to each other is called a parallelogram.

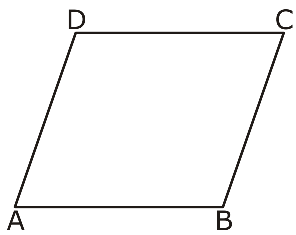


So if AB||CD and AD||BC, then ABCD is a parallelogram and in short we write it as ||gm ABCD.

3. **Rectangle.** A ||gm with one of its angle as 90^o is called a rectangle. Here ABCD is a rectangle with AB || DC, AD||BC and $\angle A = 90^o$ and in short we write it as rect. ABCD.



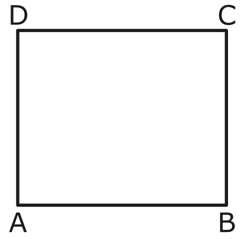
4. **Rhombus.** A quadrilateral with all its sides equal is called a rhombus.



In the adjacent figure AB = BC = CD = DA.

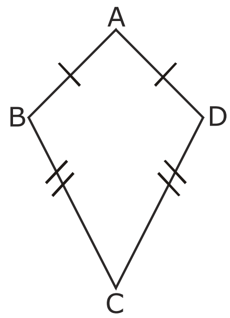
$\therefore$ ABCD is a rhombus and in short we write it as 'rhomb. ABCD'.

5. **Square .** A quadrilateral with all its sides equal and with all of its angles as 90º is called a square. Here in the figure, AB = BC = CD = DA and $\angle A = \angle B = \angle C = \angle D = \angle 90º$.



∴ ABCD is a square and in short we write it 'sq. ABCD.

6. **Kite.** A quad. is said to be in the shape of a kite if its two pairs of adjacent sides are equal.



Here ABCD is a kite with AB = AD and BC = CD.

Thus we have seen that rectangle, square, rhombus are all some special types of parallelograms, but trapezium and kite are not parallelograms.

A square is a rectangle but every rectangle is not a square.

A parallelogram is a trapezium but every trapezium is not a parallelogram.

Properties of a parallelogram.

In a parallelogram, each of the following properties hold :

(i) Diagonals of a ||gm divide it into two congruent triangles.

(ii) Opposite sides of a parallelogram are equal.

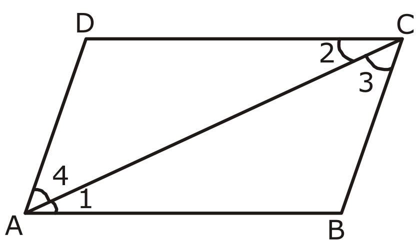
(iii) Opposite angles of a parallelogram are equal.

(iv) Diagonals of a parallelogram bisect each other.

All above necessary properties of a ||gm can be proved in the form of theorems as discussed below-

Theorem 2. A diagonal of a parallelogram divides it into two congruent triangles (CBSE 2010) (NCERT Example)

Given. A parallelogram ABCD with diagonal AC.



To prove. $\Delta ABC \cong \Delta CDA$.

Proof. Since ABCD is a parallelogram,

∴ AB || DC and AD || BC.

Now in ΔABC and ΔCDA,

AB || DC and AC is transversal,

∴ $\angle 1 = \angle 2$ (alt. int. $\angle$'s)

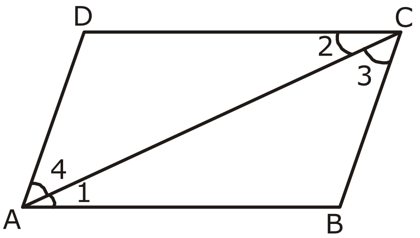
Also AD || BC and AC is transversal,

∴ $\angle 3 = \angle 4$ (alt. int. $\angle$'s)

and AC = CA (common)

∴ $\Delta ABC \cong \Delta CDA$ (ASA congruence condition.)

Theorem 3. In a parallelogram opposite sides are equal.



Given. A parallelogram ABCD.

To Prove. AB = DC and AD = BC.

Construction. Join AC.

Proof. As ABCD is a ||gm, AB||DC and AD||BC.

Now AB||DC and AC is transversal,

∴ $\angle 1 = \angle 2$ (alt. int. $\angle$'s)

Also AD || BC and AC is transversal,

∴ $\angle 3 = \angle 4$ (alt. int. $\angle$'s)

and AC = CA (common)

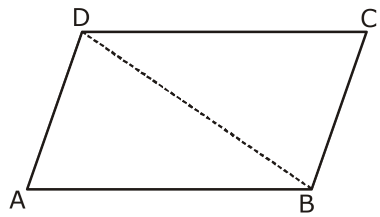
∴ $\Delta ABC \cong \Delta CDA$ (ASA congruence condition.)

∴ AB = CD and BC = DA (cpct).

Theorem 4. In a parallelogram opposite angles are equal.

Given. A ||gm ABCD.

To prove. $\angle A = \angle C$ and $\angle B = \angle D$.



Construction. Join BD.

Proof. In ΔABD and $\angle BCD$,

AB = DC

and AD = BC

($\because$ opposite sides of a ||gm are equal)

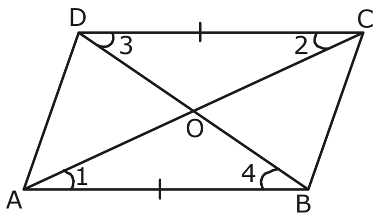
BD = BD (Common)

∴ $\angle BDA = \angle BCD$ (cpct)

Similarly, we can prove that $\angle ABC = \angle ADC$,

Theorem 5. In a parallelogram diagonals bisect each other .

Given. A ||gm ABCD with diagonals AC and BD intersecting each other at O.



To prove. OA = OC and OB = OD.

Proof. In ΔAOB and ΔCOD,

AB = CD (opposite sides of a ||gm)

$\because$ AB||CD and AC transversal

∴ $\angle 1 = \angle 2$ (alt. int. $\angle$'s)

$\because$ AB||CD and BD transversal

∴ $\angle 3 = \angle 4$ (alt. int. $\angle$'s)

∴ $\Delta AOB \cong \Delta COD$ (ASA congruence condition)

∴ AB = CO and BO = DO (cpct).

SOLVED PROBLEMS

Ex.1 Three angles of a quadrilateral measure 56°, 100° and 88°. Find the measure of the fourth angle.

Sol. Let the measure of the fourth angle be x.

$\therefore$ $56° + 100° + 88° + x° = 360°$ $\therefore$ [Sum of all the angles of quadrileteral is 360°]

$\Rightarrow$ $244°+x=360°$

$\Rightarrow$ $x = 360° - 244° = 116°$

Hence, the measure of the fourth angle is 116°.

Ex.2 The angles of a quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the quadrilateral.

Sol. Let the four angles of the quadrilateral be 3x, 5x, 9x and 13x . **[NCERT]**

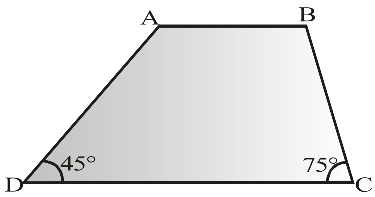
$\therefore$ $3x + 5x + 9x + 13x = 360°$ $\therefore$ [Sum of all the angles of quadrileteral is 360°]

$\Rightarrow$ $30x = 360°$

$\Rightarrow$ $x = 12°$

Hence, the angles of the quadrilateral are $3 \times 12° = 36°$, $5 \times 12° = 60°$, $9 \times 12° = 108°$ and $13 \times 12° = 156°$.

Ex.3 In figure, ABCD is a trapezium in which AB||CD. If $\angle D = 45°$ and $\angle C = 75°$, find $\angle A$ and $\angle B$.



Sol. We have, AB || CD and AD is a transversal.

so, $\angle A + \angle D = 180°$ [Interior angles on the same side of the transversal]

$\Rightarrow$ $\angle A + 45° = 180°$ [$\because \angle D = 45°$]

$\Rightarrow$ $\angle A = 180° - 45° = 135°$

Similarly, AB||CD and BC is a transversal.

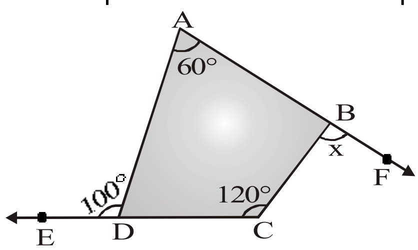
so, $\angle B + \angle C = 180°$

$\Rightarrow$ $\angle B + 75° = 180°$ [$\because \angle C = 75°$]

$\Rightarrow$ $\angle B = 105°$

Hence $\angle A = 135°$ and $\angle B = 105°$

Ex.4 In the given figure, sides AB and CD of the quadrilateral ABCD are produced. Find the value of x.



Sol. Since, $\angle ADE + \angle ADC = 180°$ [Linear pair]

$\Rightarrow$ $100° + \angle ADC = 180°$ [$\because \angle ADE = 100°$]

$\Rightarrow$ $\angle ADC = 180° - 100° = 80°$

In quadrilateral ABCD

$\angle ADC + \angle A + \angle ABC + \angle C = 360°$ $\therefore$ [Sum of all the angles of quadrileteral is 360°]

$\Rightarrow$ $80° + 60° + \angle ABC + 120° = 360°$

$\Rightarrow$ $\angle ABC + 260° = 360°$

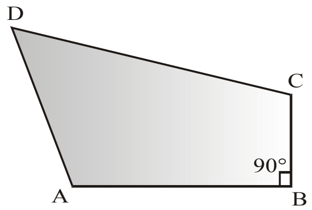
$\Rightarrow$ $\angle ABC = 100°$

But, $\angle ABC + x = 180°$ [Linear pair]

$\therefore$ $100° + x = 180°$

Hence, x = 80°.

Ex.5 In quadrilateral ABCD $\angle B = 90°$, $\angle C - \angle D = 60°$ and $\angle A - \angle C - \angle D = 10°$. Find $\angle A$, $\angle C$ and $\angle D$.



Sol. $\angle A + \angle B + \angle C + \angle D = 360°$ (Sum of the four angles of a quadrilateral is 360°)

$\Rightarrow \quad \angle A + \angle C + \angle D = 360° - \angle B$

$\angle A + \angle C + \angle D = 360° - 90°$

$\angle A + \angle C + \angle D = 270° \quad ...(1)$

It is given that

$\angle A - \angle C - \angle D = 10° \quad ...(2)$

$\angle C - \angle D = 60° \quad ...(3)$

Adding (1) and (2), we get

$(\angle A + \angle C + \angle D) + (\angle A - \angle C - \angle D) = 270° + 10°$

$\angle A + \angle C + \angle D + \angle A - \angle C - \angle D = 280°$

$2\angle A = 280°$

$\angle A = \frac{280°}{2} \quad \Rightarrow \quad \angle A = 140°$

From (1), $140° + \angle C + \angle D = 270°$

$\Rightarrow \quad \angle C + \angle D = 270° - 140°$

$\Rightarrow \quad \angle C + \angle D = 130° \quad ...(4)$

Adding (3) and (4), we get

$(\angle C - \angle D) + \angle C + \angle D = 60° + 130°$

$\angle C - \angle D + \angle C + \angle D = 190°$

$2 \times \angle C = 190°$

$\angle C = \frac{190°}{2} \quad \Rightarrow \quad \angle C = 95°$

Subtracting (3) from (4), we get

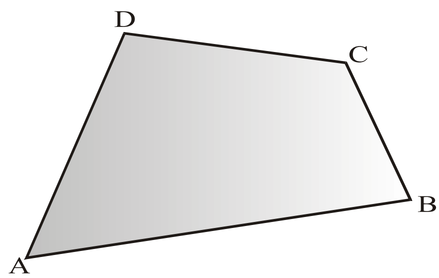
$(\angle C + \angle D) - (\angle C - \angle D) = 130° - 60°$

$\angle C + \angle D - \angle C + \angle D = 70°$

$2 \times \angle D = 70°$

$\angle D = \frac{70°}{2} \quad \Rightarrow \quad \angle D = 35°$

Ex.6 In quadrilateral ABCD $\angle A + \angle C = 140°$, $\angle A : \angle C = 1 : 3$ and $\angle B : \angle D = 5 : 6$. Find the $\angle A$, $\angle B$, $\angle C$ and $\angle D$.



Sol. $\angle A + \angle C = 140°$ (Given)

$\angle A : \angle C = 1 : 3$ (Given)

sum of ratio = 1 + 3 = 4

$\Rightarrow \quad \angle A = \frac{1}{4} \times 140° = 35°$

and $\quad \angle C = \frac{3}{4} \times 140° = 35° \times 3 = 105°$

Sum of all the angles of quadrilateral is 360°

We have $\angle A + \angle B + \angle C + \angle D = 360°$

$\Rightarrow \quad 35° + \angle B + 105° + \angle D = 360°$

$\Rightarrow \quad \angle B + \angle D + 140° = 360°$

$\Rightarrow \quad \angle B + \angle D = 360° - 140°$

$\Rightarrow \quad \angle B + \angle D = 220°$

It is given that,

$\angle B : \angle D = 5 : 6$

sum of ratios = 5 + 6 = 11

$\Rightarrow \quad \angle B = \frac{5}{11} \times 220° = 20° \times 5 = 100°$

and $\quad \angle D = \frac{6}{11} \times 220° = 20° \times 6 = 120°$

Hence, $\angle A = 35°$, $\angle B = 100°$, $\angle C = 105°$ and $\angle D = 120°$

Ex.7 Prove that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

Sol. **Given:** ABCD is a quadrilateral where diagonals AC and BD meet at 0, such that AO = OC, OB = OD and $AC \perp BD$ **[NCERT]**

To Prove: Quadrilateral ABCD is a rhombus, i.e., AB = BC = CD = DA

Proof : In ΔAOB and ΔAOD, OB=OD [Common]

AO=AO [Given]

$\angle AOB = \angle AOD$ [Each = 90°]

$\therefore \quad \Delta AOB \cong \Delta AOD$ [SAS Rule]

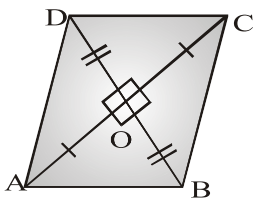
$\therefore \quad AB = AD$ [C.P.C.T.]

Similarly, we can prove that

AB = BC ...(i)

BC = CD ...(ii)

CD = AD ...(iii)



From (i), (ii), (iii) and (iv), we obtain

AB = BC = CD = DA

$\therefore$ Quadrilateral ABCD is a rhombus.

Ex.8 Prove that the diagonals of a square are equal and bisect each other at right angles. **[NCERT]**

Sol. **Given:** ABCD is a square.

To Prove: (i) AC = BD (ii) AC and BD bisect each other at right angles.

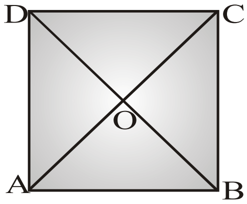
Proof: In ΔABC and ΔBAD,

AB = BA [Common]

BC = AD [Opp. sides of square ABCD]

$\angle ABC = \angle BAD$ [Each = 90° ($\because$ ABCD is a square] $\therefore$ $\Delta ABC \cong \Delta BAD$ [SAS Rule]

$\therefore$ AC = BD [C.P.C.T.] ... (i)



In ΔAOD and ΔBOC

AD = CB [Opp. sides of square ABCD]

$\angle OAD = \angle OCB$ [Alternate angles as AD||BC and transversal AC intersects them]

$\angle ODA = \angle OBC$ [Alternate angles as AD||BC and transversal BD intersects them]

$\Delta AOD \cong \Delta BOC$ [ASA Rule]

$\therefore$ OA = OC and OB = OD [C.P.C.T.] ...(ii)

So, O is the mid point of AC and BD.

Now, In ΔAOB and ΔCOB

AB = BC [Given]

OA = OC [from (ii)]

OB = OB [Common]

$\therefore$ $\Delta AOB \cong \Delta COB$ [By SSS Rule]

$\therefore$ $\angle AOB = \Delta BOC$ [C.P.C.T]

But $\angle AOB + \angle BOC = 180°$ [Linear pair]

$\angle AOB + \angle AOB = 180°$

[AOB = BOC proved earlier]

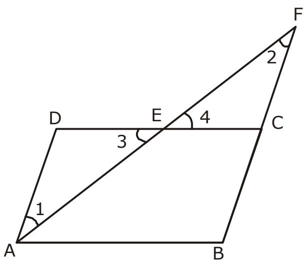
$\Rightarrow$ $2\angle AOB = 180°$

$\Rightarrow$ $\angle AOB = \frac{180°}{2} = 90°$

$\therefore$ $\angle AOB = \angle BOC = 90°$

$\therefore$ AC and BD bisect each other at right angles.

Ex.9 In the adjacent figure ABCD is a parallelogram. E is the mid point of CD. AE is joined and produced to meet BC at F. Show that BF = 2AD.



Sol. In ΔADE and ΔFCE,

$\because$ AD||BF and AF transversal

$\therefore$ $\angle 1 = \angle 2$ (alt. int. $\angle$'s)

$\angle 3 = \angle 4$ (vert. opp. $\angle$'s

DE = EC ($\because$ E is mid point of CD)

$\therefore$ $\Delta ADE \cong \Delta FCE$ (AAS congruence condition)

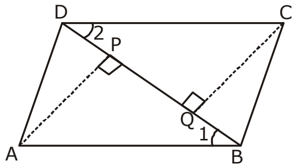
$\therefore$ AD = CF (cpct)

But AD = BC (opp. sides of a ||gm)

$\Rightarrow$ AD + AD = BC + CF

$\Rightarrow$ 2AD = BF.

Ex.10 ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD. Show that : (i) ΔAPB $\cong \Delta$ CQD, (ii) AP = CQ. **[NCERT]**



Sol. In Δ APB and Δ CQD ,

$\because$ AB ||DC and BD transversal

$\therefore$ $\angle 1 = \angle 2$ (alt. int. $\angle$'s)

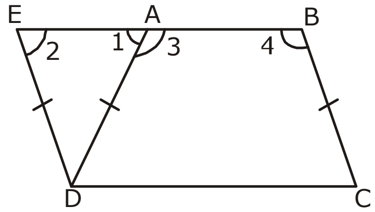
AB = DC (opp. sides of a ||gm)

$\angle$APB = $\angle$CQD (each 90º)

$\therefore$ Δ APB $\cong$ Δ CQS (AAS congruence condition)

$\therefore$ AP = CQ (cpct).

Ex.11 In the given figure ABCD is an isosceles trapezium with AB||DC and AD = BC. Prove that $\angle$A = $\angle$B.



[NCERT]

Sol. Let us draw DE||BC, which meets BA produced at E.

Now since AB||DC $\therefore$ EB||DC and ED ||BC (by construction)

$\therefore$ BCDE is a parallelogram.

$\therefore$ BC = De (opp. sides of a||gm)

But BC = AD (given)

$\therefore$ AD = DE

$\Rightarrow$ ¶$\angle 1 = \angle 2$

(angles opp. to equal sides are equal)

Now $\angle 1 + \angle 3$ = 180º (Linear pair)

and $\angle 2 + \angle 4$ = 180º (co. int. $\angle$'s)

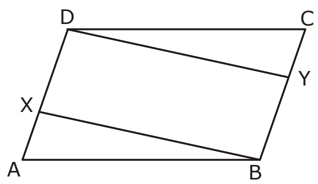
$\therefore$ $\angle 1 + \angle 3 = \angle 2 + \angle 4$

$\Rightarrow$ $\angle 1 = \angle 4$ (using $\angle 1 = \angle 2$)

or $\angle$A = $\angle$B.

Ex.12 ABCD is a parallelogram. X and Y are points on sides AD and BC such that AX = $\frac{1}{3}$BC and CY = $\frac{1}{3}$ BC.

Prove that BXDY is a parallelogram.



Sol. ABCD is a parallelogram.

$\therefore$ AD||BC (opp. sides of a||gm)

$\Rightarrow$ XD||BY ...(1)

Also, AD = BC (opp. sides of a||gm are equal)

⇒ $\frac{1}{3}AD - \frac{1}{3}BC$

⇒ AX = CY

$\left(\because \text{given that } AX = \frac{1}{3}AD \text{ and } CY = \frac{1}{3}BC\right)$

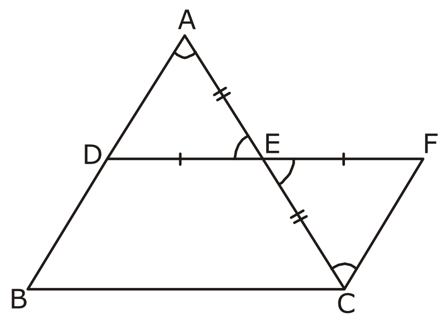
⇒ AD – AX = BC – CY

⇒ XD = BY ...(2)

From equations (1) and (2), we get XBYD is a ||gm.

(if in a quad. a pair of opp. sides is equal and parallel then it is a||gm)

Ex.13 In ΔABC, D and E are mid points of sides AB and AC respectively. DE is joined and produced till F such that DE = EF. CF is joined. Show that BCFD is a parallelogram.



Sol. In ΔADE and ΔCFE

AE = EC (∵ E is the mid point of AC)

DE = EF (by construction)

∠AED = ∠CEF (vert. int. ∠'s)

∴ ΔADE ≅ ∠CFE (SAS congruence condition)

∴ ∠FCE = ∠EAD (cpct)

But it is a pair of alt. int. ∠'s, therefore CF||AD

or CF||BD (∵ BD and AD are coincident lines)

Also CF = AD (cpct)

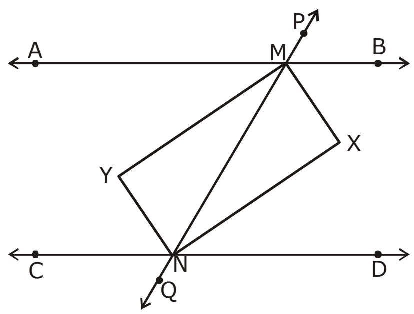
But Ad = BD (∵ D is mid point of AB)

∴ CF = BD

Hence in quad. BCFD, CF||BD and CF = BD.

∴ BCFD is a parallelogram.

Ex.14 Two parallel lines l and m are intersected by a transversal p. Show that the quadrilateral formed by the bisectors of interior angles is a rectangle. **[NCERT]**



Sol. Let two parallel lines l and m be intersected by a transversal p at M and N respectively. The bisectors of $\angle BMN$ and $\angle DNM$ intersect each other at X and bisectors of $\angle AMN$ and $\angle CNM$ intersect each other at Y. We are required to show that MXNY is a rectangle.Now since AB||CD and PQ transversal, therefore

$\angle BMN = \angle MNC$ (alt. int. $\angle$'s)

$\Rightarrow \quad \frac{1}{2}\angle BMN = \frac{1}{2}\angle MNC$

$\Rightarrow \quad \angle XMN = \angle MNY$

($\because$ MX bisects $\angle BMN$ and NY bisects $\angle MNC$)

But these form a pair of alternate angles for lines MX and NY with MN transversal.

$\therefore \quad$ MX || NY

Similarly we can prove that MY||NX.

$\because$ Both the pairs of opposite sides are parallel,

$\therefore \quad$ MXNY is a parallelogram.

Also, $\angle BMN + \angle MND = 180^\circ$ (Co-interior $\angle$'s)

$\Rightarrow \quad \frac{1}{2}\angle BMN + \frac{1}{2}\angle MND = 90^\circ$

$\Rightarrow \quad \angle XMN + \angle XNM + \angle MXN = 180^\circ$

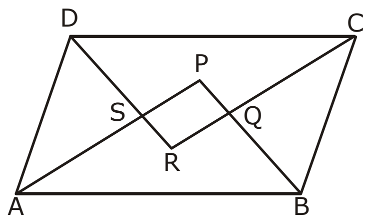
$\Rightarrow \quad 90^\circ + \angle MXN = 180^\circ$ (using eqn. (1))

$\Rightarrow \quad \angle MXN = 90^\circ$

Hence MXYN is a||gm with one of its angle as 90º, So it is a rectangle.

Ex.15 Show that the bisectors of angles of a parallelogram form a rectangle.

Sol. Let ABCD be a ||gm. Let bisectors of $\angle A$ and $\angle B$ intersect each other at P, bisectors of $\angle B$ and $\angle C$ intersect each other at Q, bisectors of $\angle C$ and $\angle D$ intersect each other at R and Bisectors of $\angle D$ and $\angle A$ intersect each other at S. We are to prove that PQRS is a rectangle.



Now since AP bisects $\angle A$,

$\therefore \quad \angle PAB = \frac{1}{2}\angle A$

and since BP bisects $\angle B$,

$\therefore \quad \angle PAB = \frac{1}{2}\angle B$

But $\angle A + \angle B = 180^\circ$ (co-interior angles)

$\therefore \quad \frac{1}{2}\angle A + \frac{1}{2}\angle B = 90^\circ$

$\Rightarrow \quad \angle PAB + \angle PBA = 90^\circ$ .....(1)

$\angle PAB + \angle PBA + \angle P = 180^\circ$

(angle sum property of triangle)

$\Rightarrow \quad 90^\circ + \angle P = 180^\circ$ (using eqn.1)

$\Rightarrow \quad \angle P = 90^\circ$

Similarly we can prove that $\angle Q$, $\angle R$, $\angle S$ each is 90º.

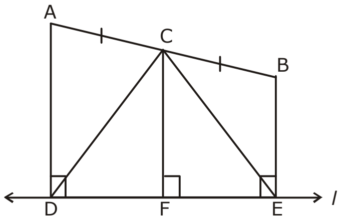
Now we have a quad. PQRS in which opposite angles are equal, so PQRS is a parallelogram.

Also $\angle P = 90^\circ$

$\therefore \quad$ PQRS is a rectangle.

($\because$ a parallelogram with any one of its angle as right angle is a rectangle)

Ex.16 In the given figure, on line I perpendicular lines AD and BE are drawn from points A and B. IF C is the mid point of AB, prove that CD = CE.



Sol. Let us draw CF perpendicular to I. Now AD, CF and BE all are perpendicular to I

AD||CF||BE

and these lines make equal intercepts on transversal

AB ($\because$ AC = BC, C being mid point of AB).

$\therefore$ Intercepts made on transversal I should also be equal i.e.,

DF = FE (by intercept theorem)

Now in ΔCDF and ΔCEF,

DF = EF (proved above)

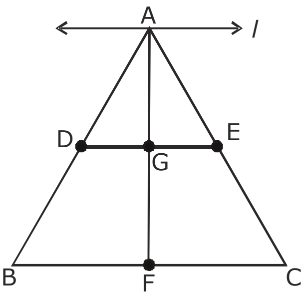
CF = CF (common)

$\angle CFD = \angle CFE$ (each 90º)

$\therefore$ $\Delta CDF \cong \Delta CEF$ (SAS congruence condition)

CD = CE (cpct).

Ex.17 In ΔABC, D and E are the mid points of sides AB and AC. F is any point of BC. Line joining A and F meets DE at G. Show that G is the mid point of AF.



Sol. Let us draw a line *l* through A, parallel to BC. Also DE||BC (by mid point theorem).

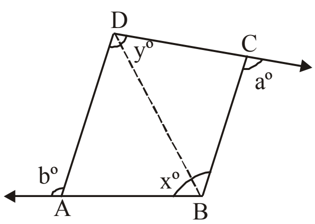
$\therefore \ell$ ||DE||BC and AB transversal on which intercepts made by these parallel lines are equal as AD = BD ($\because$ given that D is the mid point of AB).

AF is another transversal, therefore by intercept theorem, intercepts made by the parallel lines on AF are also equal i.e., AG = GF.

or G is the mid point of AF.

Ex.18 The sides BA and DC of a quadrilateral ABCD are produced as shown in fig. Prove that a + b = x + y.

Sol. Join BD. In ΔABD, we have



$\angle ABD + \angle ADB = b^o$ ....(i)

In ΔCBD, we have

$\angle CBD + \angle CDB = a^o$ ....(ii)

Adding (i) and (ii), we get

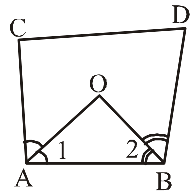
$(\angle ABD + \angle CBD) + (\angle ADB + \angle CDB) = a^o + b^o$

$\Rightarrow$ $x^o + y^o = a^o + b^o$

Hence, x + y = a + b

Ex.19 In a quadrilateral ABCD, AO and BO are the bisectors of $\angle A$ and $\angle B$ respectively. Prove that $\angle AOB = \frac{1}{2}(\angle C + \angle D)$.

Sol. In ΔAOB, we have



$\angle AOB + \angle 1 + \angle 2 = 180^\circ$

$\Rightarrow \quad \angle AOB = 180^\circ - (\angle 1 + \angle 2)$

$\Rightarrow \quad \angle AOB = 180^\circ - \left(\frac{1}{2}\angle A + \frac{1}{2}\angle B\right) \qquad \left[\because \angle 1 = \frac{1}{2}\angle A \text{ and } \angle 2 = \frac{1}{2}\angle B\right]$

$\Rightarrow \quad \angle AOB = 180^\circ - \frac{1}{2}(\angle A + \angle B)$

$\Rightarrow \quad \angle AOB = 180^\circ - \frac{1}{2}[360^\circ - (\angle C + \angle D)]$

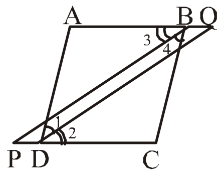
$[\because \angle A + \angle B + \angle C + \angle D = 360^\circ$

$\therefore \quad \angle A + \angle B = 360^\circ - (\angle C + \angle D)]$

$\Rightarrow \quad \angle AOB = 180^\circ - 180^\circ + \frac{1}{2}(\angle C + \angle D)$

$\Rightarrow \quad \angle AOB = \frac{1}{2}(\angle C + \angle D)$

Ex.20 In figure bisectors of $\angle B$ and $\angle D$ of quadrilateral ABCD meet CD and AB produced at P and Q respectively. Prove that $\angle P + \angle Q = \frac{1}{2}(\angle ABC + \angle ADC)$



Sol. In ΔPBC, we have

$\therefore \quad \angle P + \angle 4 + \angle C = 180^\circ$

$\Rightarrow \quad \angle P + \frac{1}{2}\angle B + \angle C = 180^\circ \quad ...(i)$

In ΔQAD, we have

$\angle Q + \angle A + \angle 1 = 180^\circ$

$\Rightarrow \quad \angle Q + \angle A + \frac{1}{2}\angle D = 180^\circ \quad ...(ii)$

Adding (i) and (ii), we get

$\angle P + \angle Q + \angle A + \angle C + \frac{1}{2}\angle B + \frac{1}{2}\angle D = 180^\circ + 180^\circ$

$\Rightarrow \quad \angle P + \angle Q + \angle A + \angle C + \frac{1}{2}\angle B + \frac{1}{2}\angle D = 360^\circ$

$\Rightarrow \quad \angle P + \angle Q + \angle A + \angle C + \frac{1}{2}(\angle B + \angle D) = \angle A + \angle B + \angle C + \angle D$

$[\therefore$ In a quadrilateral ABCD

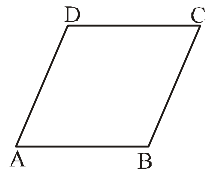
$\angle A + \angle B + \angle C + \angle D = 360^\circ]$

$\Rightarrow \quad \angle P + \angle Q = \frac{1}{2}(\angle B + \angle D)$

$\Rightarrow \quad \angle P + \angle Q = \frac{1}{2}(\angle ABC + \angle ADC)$

Ex.21 In a parallelogram ABCD, prove that sum of any two consecutive angles is 180º.

Sol. Since ABCD is a parallelogram. Therefore, AD || BC.



Now, AD || BC and transversal AB intersects them at A and B respectively.

$\therefore$ $\angle A + \angle B = 180^\circ$

[$\because$ Sum of the interior angles on the same side of the transversal is 180º]

Similarly, we can prove that

$\angle B + \angle C = 180^\circ$, $\angle C + \angle D = 180^\circ$ and $\angle D + \angle A = 180^\circ$.

Ex.22 In a parallelogram ABCD, $\angle D = 115^\circ$, determine the measure of $\angle A$ and $\angle B$.

Sol. Since the sum of any two consecutive angles of a parallelogram is 180º. Therefore,

$\angle A + \angle D = 180^\circ$ and $\angle A + \angle B = 180^\circ$

Now, $\angle A + \angle D = 180^\circ$

$\Rightarrow$ $\angle A + 115^\circ = 180^\circ$

[$\because \angle D = 115^\circ$ (given)]

$\Rightarrow$ $\angle A = 65^\circ$

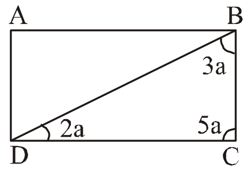
and $\angle A + \angle B = 180^\circ$

$\Rightarrow$ $65^\circ + \angle B = 180^\circ$

$\Rightarrow$ $\angle B = 115^\circ$

Thus, $\angle A = 65^\circ$ and $\angle B = 115^\circ$

Ex.23 In figure find the four angles A, B, C and D in the parallelogram ABCD.



Sol. In ΔBCD we have

$\angle BDC + \angle DCB + \angle CBD = 180^\circ$

$\Rightarrow$ $2a + 5a + 3a = 180^\circ$

$\Rightarrow$ $10a = 180^\circ$

$\Rightarrow$ $a = 18^\circ$

$\therefore$ $\angle C = 5a = 5 \times 18^\circ = 90^\circ$

Since opposite angles are equal in a parallelogram. Therefore,

$\angle A = \angle C \quad \Rightarrow \quad \angle A = 90^\circ$

Since the sum of the angles of a quadrilateral is 360º. Therefore,

$\angle A + \angle B + \angle C + \angle D = 360^\circ$

$\Rightarrow$ $2(\angle A + \angle B) = 360^\circ$

[$\because$ $\angle A = \angle C$ and $\angle B = \angle D$]

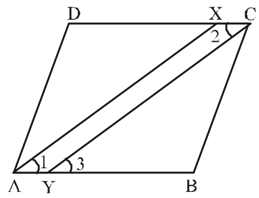
$\Rightarrow$ $\angle A + \angle B = 180^\circ$

$\Rightarrow$ $90^\circ + \angle B = 180^\circ$ [$\because$ $\angle A = 90^\circ$]

$\Rightarrow$ $\angle B = 90^\circ$

Hence, $\angle A = 90^\circ$, $\angle B = 90^\circ$, $\angle C = 90^\circ$ and $\angle D = 90^\circ$

Ex.24 ABCD is a parallelogram and line segments AX & CY are bisector of $\angle A$ and $\angle C$. Show that AX || CY.



Sol. Since opposite angles are equal in a parallelogram. Therefore, in parallelogram ABCD, we have

$\angle A = \angle C$

$\Rightarrow \quad \frac{1}{2}\angle A = \frac{1}{2}\angle C$

$\Rightarrow \quad \angle 1 = \angle 2 \quad$ ....(i)

[$\because$ AX and CY are bisectors of $\angle A$ and $\angle C$ respectively]

Now, AB || DC and the transversal CY intersects them.

$\therefore \quad \angle 2 = \angle 3 \quad$ ....(ii)

[$\because$ Alternate interior angles are equal]

From (i) and (ii), we get

$\angle 1 = \angle 3$

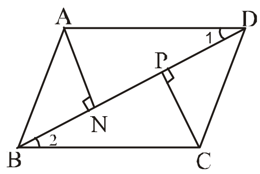
Thus, transversal AB intersects AX and YC at A and Y such that $\angle 1 = \angle 3$

i.e. corresponding angles are equal.

$\therefore \quad$ AX || CY

Ex.25 In figure AN and CP are perpendiculars to the diagonal BD of a parallelogram ABCD. Prove that:

(i) $\Delta ADN \cong \Delta CBP$ (ii) AN = CP **[NCERT]**



Sol. Since ABCD is a parallelogram.

$\therefore \quad$ AD || BC

Now, AD || BC and transversal BD intersects them at B and D.

$\therefore \quad \angle 1 = \angle 2$

[$\because$ Alternate interior angles are equal]

Now, in Δs ADN and CBP, we have

$\angle 1 = \angle 2$

$\angle AND = \angle CPD$ and, AD = BC

[$\because$ Opposite sides of a $||^{gm}$ are equal]

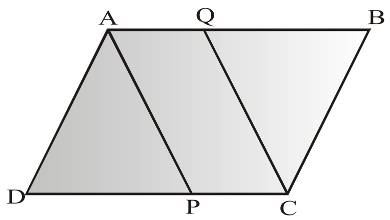
So, by AAS criterion of congruence

$\Delta ADN \cong \Delta CBP$

AN = CP

[$\because$ Corresponding parts of congruent triangles are equal]

Ex.26 In figure ABCD is a parallelogram and AP and CQ are bisectors of $\angle A$ and $\angle C$. Prove that AP||CQ.



Sol. We have $\angle A = \angle C$ [Opposite angles of a ||^{gm}]

$\frac{1}{2}\angle A = \frac{1}{2}\angle C$

$\Rightarrow$ $\angle PAQ = \angle PCQ$ ...(1)

Now, AB || CD and CQ is a transversal. Therefore ,

$\angle PCQ = \angle BQC$ [Alternate angles] ...(2)

$\Rightarrow$ $\angle PAQ = \angle BQC$ [From (1)]

But, these are corresponding angles formed when AP and CQ are intersected by transversal AB.

$\therefore$ AP||CQ

Hence proved.

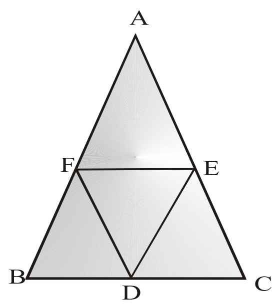
Ex.27 In the following figure, D, E and F are respectively the mid-points of sides BC, CA and AB of an equilateral triangle ABC. Prove that ΔDEF is also an equilateral triangle. **[NCERT]**

Sol. **Given :** D, E and F are respectively the mid-points of sides BC, CA and AB of an equilateral triangle ABC.

To prove : ΔDEF is also an equilateral triangle.

Proof : since the segment joining the mid points of two sides of a triangle is half of the third side. Therefore D and E are the mid point of BC and AC respectively.

$\therefore$ $DE = \frac{1}{2} AB$ ....(i)



E and F are the mid point of AC and AB respectively

$\therefore$ $EF = \frac{1}{2} BC$ ...(ii)

F and D are the mid point of AB and BC respectively

$\Rightarrow$ $FD = \frac{1}{2} AC$ ...(iii)

$\because$ ΔABC is an equilateral triangle

$\Rightarrow$ AB = BC = CA

$\Rightarrow$ $\frac{1}{2}AB = \frac{1}{2}BC = \frac{1}{2}CA$

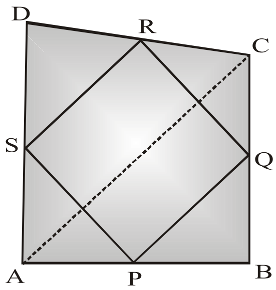
$\Rightarrow$ DE = EF = FD

using (i), (ii) & (iii)

Hence, ΔDEF is an equilateral triangle.

Hence Proved

Ex.28 ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA. AC is a diagonal. **[NCERT]**



Show that :

(i) SR || AC and SR = $\frac{1}{2}$ AC (ii) PQ = SR

(iii) PQRS is a parallelogram.

Sol. **GIVEN :** ABCD is a quadrilateral in which P, Q, R and S are mid-points of AB, BC, CD and DA. AC is a diagonal.

TO PROVE :

(i) SR || AC and SR = $\frac{1}{2}$AC

(ii) PQ = SR

(iii) PQRS is a parallelogram.

PROOF : (i) In ΔDAC,

$\because$ S is the mid-point of DA and R is the mid-point of DC

$\therefore$ SR || AC and SR = $\frac{1}{2}$ AC [By Mid-point theorem]

(ii) In ΔBAC,

$\because$ P is the mid-point of AB and Q is the mid-point of BC

$\therefore$ PQ || AC and PQ = $\frac{1}{2}$ AC [By Mid-point theorem]

But from (i) SR = $\frac{1}{2}$AC & (ii) PQ = $\frac{1}{2}$ AC

$\Rightarrow$ PQ = SR

(iii) PQ || AC [From (ii)]

SR || AC [From (i)]

$\therefore$ PQ || SR

[Two lines parallel to the same line are parallel to each other]

Also, PQ = SR [From (ii)]

$\therefore$ PQRS is a parallelogram.

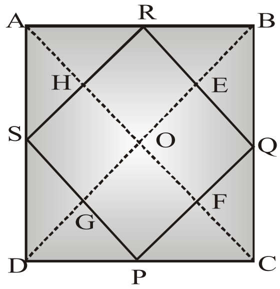
[A quadrilateral is a parallelogram if a pair of opposite sides is parallel and is of equal length]

Ex.29 Show that the quadrilateral formed by joining the mid-point of the consecutive sides of a square is also a square. **[NCERT]**

Sol. **Given :** ABCD is a square. R, Q, P and S are the mid-points of the consecutive isdes AB, BC, CD and DA respectively. PQ, QR, RS and SP are joined.

To prove : Quadrilateral PQRS is a square.

Construction : Join AC and BD



Proof : RQ || AC and RQ = $\frac{1}{2}$ AC ... (1)

SP || AC and SP = $\frac{1}{2}$ AC ...(2)

From (1) & (2)

$\therefore$ RQ = SP and RQ || SP

Similarly, SR = PQ and SR || PQ

$\therefore$ PQRS is a parallelogram

$\because$ RQ || AC $\therefore$ RE || HO

$\because$ SR || PQ $\therefore$ HR || OE

$\therefore$ OERH is a parallelogram.

$\therefore$ $\angle$R = $\angle$HOE [Opposite $\angle$s of a || gm]

But $\angle$HOE = 90° [Diagonal of square bisect at 90°]

$\therefore$ $\angle$R = 90° $\therefore$ Quadrilateral PQRS is a rectangle.

But AC = BD [Diagonal of a square are equal]

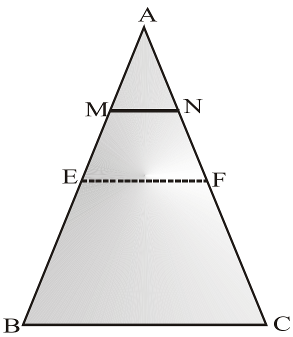
$\therefore$ HF = GE or PQ = QR, so all sides are equal

$\therefore$ PQ = QR = RS = SP $\therefore$ Quadrilateral PQRS is a square.

Hence Proved.

Ex.30 In triangle ABC, points M and N on sides AB and AC respectively are taken so that AM = $\frac{1}{4}$AB and AN = $\frac{1}{4}$AC. Prove that MN = $\frac{1}{4}$BC.

Sol. **Given :** In triangle ABC, points M and N on the sides AB and AC respectively are taken so that AM = $\frac{1}{4}$AB and AN = $\frac{1}{4}$AC.



To prove : MN = $\frac{1}{4}$BC.

Construction : Join EF where E and F are the mid points of AB and AC respectively.

Proof : $\because$ E is the mid-point of AB and F is the mid-point of AC.

$\therefore$ EF || BCand EF = $\frac{1}{2}$BC ...(1)

Now, AE = $\frac{1}{2}$AB and AM = $\frac{1}{4}$AB

$\therefore$ AM = $\frac{1}{2}$AE

Similarly, AN = $\frac{1}{2}$AF

$\Rightarrow$ M and N are the mid-points of AE and AF resepctively.

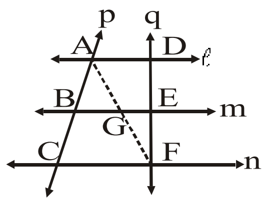
$\therefore$ MN || EF and MN = $\frac{1}{2}$EF = $\frac{1}{2}\left(\frac{1}{2}BC\right)$ [From (1)]

MN = $\frac{1}{4}$BC.

Hence Proved.

Ex.31 In figure ℓ,m and n are three parallel lines intersected by transversals p and q such that ℓ, m and n cut-off equal intercepts AB and BC on p. Show that ℓ, m and n cut off equal intercepts DE and EF on q also.

Sol.



Given : AB = BC

To prove : DE = EF

Construction : Join AF, it intersect line m at G

In ΔACF, B is the mid-point of AC ($\because$ AB = BC and BG || CF. Therefore, G is the mid-point of AF.

In ΔAFD, G is the mid-point of AF and GD || AD.

$\therefore$ E is the mid-point of DE

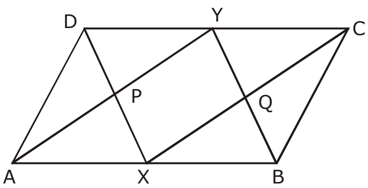
$\Rightarrow$ DE = EF

Hence, ℓ, m and n cut of equal intercepts DE and EF on q.

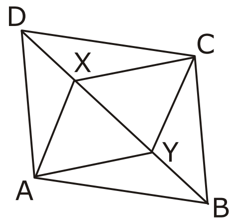
EXERCISE – I | UNSOLVED PROBLEMS

Q.1 The angles of a quadrilateral are in ratio 1 : 2 : 3 : 4. Find all the four angles of the quadrilateral.

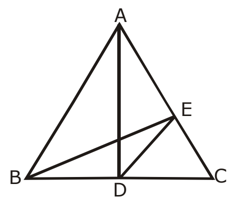
Q.2 In the given figure, X, Y are the mid-points of sides AB and CD of a parallelogram ABCD. AY and DX are intersecting at P, CX and BY are intersecting at Q. Show that PXQY is a parallelogram. **[NCERT]**



Q.3 In the given figure, ABCD is a parallelogram and X and Y are point on the diagonal BD such that DX = BY. Prove that AXCY is a parallelogram.

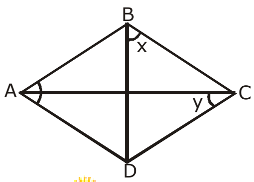


Q.4 In the given figure, AD is median and DE || AB. Prove that BE is the median.

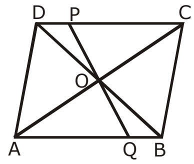


Q.5 Prove that the figure formed by joining the midpoint of the pairs of consecutive sides of a quadrilateral is a parallelogram.

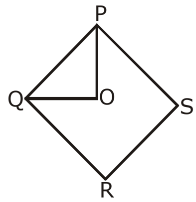
Q.6 ABCD is a rhombus with one $\angle BAD = 50^\circ$, find $\angle x$ and $\angle y$ in given figure.



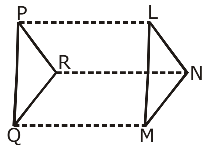
Q.7 In the given figure, ABCD is a parallelogram whose diagonals intersect each other at O. Through O, PQ is drawn then prove that OP = OQ.



Q.8 In the given figure, PQRS is a quadrilateral PO and QO are bisectors of $\angle P$ and $\angle Q$ respectively, then prove that $\angle QOP = \frac{1}{2}(\angle R + \angle S)$

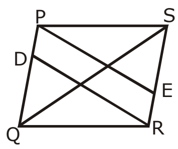


Q.9 If ΔPQR and ΔLMN be two triangles given in such away that PQ || LM, PQ = LM and QR = MN and QR || MN, then show that PR || LN and PR = LN.



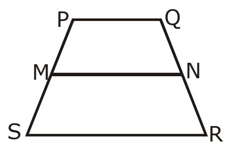
Q.10 Show that the line segment joining the mid-points of the sides of quadrilateral bisect each other.

Q.11 PQRS is a parallelogram D is a point that $PD = \frac{1}{3}PQ$ and E is a point such that $RE = \frac{1}{3}RS$. Show that quadrilateral PDRE is a parallelogram.



Q.12 M and N are the mid-points of non parallel sides of a trapezium PQRS. Prove that

(a) MN || PQ (b) $MN = \frac{1}{2}(PQ + RS)$



Q.13 Show that the four triangles formed by joining the mid-point of the three sides of a triangle are congruent to each other.

Q.14 Define the following terms :

(i) parallelogram (ii) rectangle

(iii) square (iv) rhombus

(v) trapezium.

Q.15 (a) State and prove angle sum property of a quadrilateral.

(b) State mid point theorem.

Q.16 Prove that, in a parallelogram :

(i) opposite sides are equal

(ii) opposite angles are equal

(iii) diagonals bisect each other.

Q.17 In a quarilateral ABCD, if $\angle B = \angle A + \angle C$ and BA = BD, prove that BC = BD.

Q.18 Bisectors of angles A and B of a quadrilateral ABCD intersect each-other at O. Prove that $\angle AOB = \frac{1}{2}(\angle C + \angle D)$.

Q.19 Show that the diagonals of a square are equal and perpendicular to each other.

Q.20 ABCD is a trapezium with AB || DC. If bisectors of $\angle ABC$ and $\angle BCD$ intersect each other at O, then prove that $\angle BOC = 90^o$.

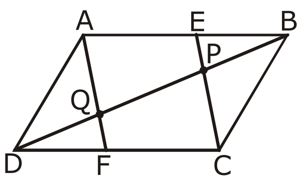
Q.21 Define an isosceles trapezium. ABCD is an isosceles trapezium with $\angle B = 60^o$ and AD || BC. Side AD is produced till E such that AE = BC, CE is joined. Find $\angle DCE$.

Q.22 Prove that the bisectors of the angles of a parallelogram enclose a rectangle.

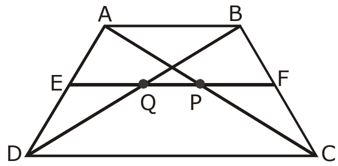
Q.23 Prove that in a triangle, the line segment joining the mid points of any two sides is parallel to the third side and is half of it.

Q.24 State and prove converse of mid point theorem.

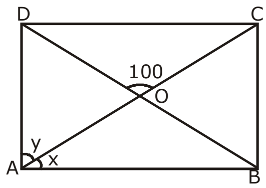
Q.25 ABCD is a parallelogram. P and Q trisect BD. AQ is joined which meets DC at F when produced also CP is joined which meets AB at E when produced. If AECF is a parallelogram, prove that E and F are the mid points of sides AB and CD respectively.



Q.26 In the given figure ABCD is a trapezium with AB || DC. E is the mid point of AD and EF || DC. EF intersects diagonals AC and BD at P and Q respectively. Prove that AC is bisected at P and BD is bisected at Q.

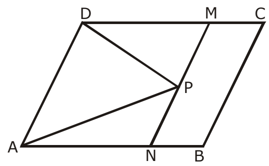


Q.27 In the given figure, ABCD is a rectangle. Find x and y.



Q.28 ABCD is a parallelogram as shown in the figure. Bisectors of angles BAD and CDA intersect each other at P. Through P a line MPN is drawn parallel to AD. Prove that :

(i) AN = DM (ii) MP = NP.



Q.29 P, Q, R are respectively the mid points of sides BC, CA and AB of ΔABC. PR and BQ intersect each other at M and PQ and CR intersect each other at N. Prove that MN || BC and $MN = \frac{1}{4} BC$.

ANSWER KEY

1. 36°,72°,108°,144°

6. x =40°, y = 50°

21. $\angle DCE = 60°$

27. x = 40°, y= 50°

EXERCISE – II SCHOOL EXAM/BOARD

Q.1 Two opposite angles of a parallelogram are $(3x - 2)°$ and $(50-x)°$. Find the measure of each angle of the parallelogram.

Q.2 If an angle of a parallelogram is two-third of its adjacent angle, find the angles of the parallelogram.

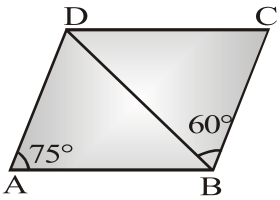
Q.3 Find the measure of all the angles of a parallelogram, if one angle is 24° less than twice the smallest angle.

Q.4 The perimeter of a parallelogram is 22 cm. If the longer side measures 6.5 cm what is the measure of the shorter side ?

Q.5 In a parallelogram ABCD, $\angle D = 135°$, determine the measures of $\angle A$, and $\angle B$.

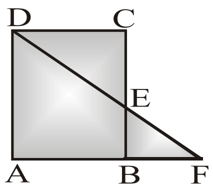
Q.6 ABCD is a parallelogram in which $\angle A = 70°$. Compute $\angle B$, $\angle C$ and $\angle D$.

Q.7 In figure ABCD is a parallelogram in which $\angle DAB = 75°$ and $\angle DBC = 60°$.



Compute $\angle CDB$ and $\angle ADB$.

Q.8 In figure ABCD is a parallelogram and E is the mid-point of side BC.



If DE and AB when produced meet at F, prove that AF = 2AB.

Q.9 In a parallelogram ABCD, determine the sum of $\angle C$ and $\angle D$.

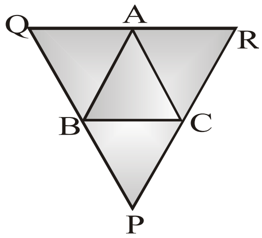
Q.10 In a parallelogram ABCD, if $\angle B = 135°$, determine the measures of its other angles.

Q.11 ABCD is a square. AC and BD intersect at O. State the measure of $\angle AOB$.

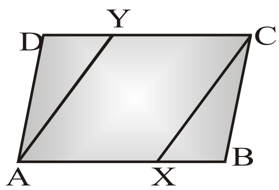
Q.12 ABCD is a rectangle with $\angle ABD = 40°$. Determine $\angle DBC$.

Q.13 The sides AB and CD of a parallelogram ABCD are bisected at E and F. Prove that EBFD is a parallelogram.

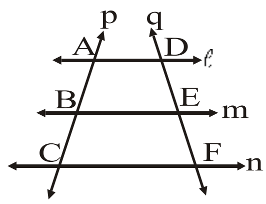
Q.14 Given ΔABC, lines are drawn through A, B and C parallel respectively to the sides BC, CA and AB forming ΔPQR. Show that $BC = \frac{1}{2} QR$.



Q.15 In the given figure, ABCD is a parallelogram and X, Y are the mid-points of the sides AB and DC respectively. Show that quadrilateral AXCY is a parallelogarm.



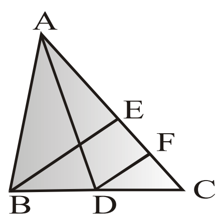
Q.16 In figure, three parallel lines ℓ, m and n are intersected by a transversal p at points A, B and C respectively and transversal q at D, E and F respectively.



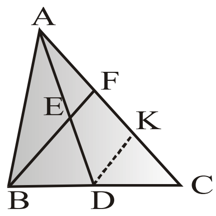
If AB : BC = 1 : 2, prove that DE : EF = 1 : 2.

Q.17 ΔABC is a triangle right angled at B; and P is the mid-point of AC.
Prove that : (i) $PQ \perp AB$ (ii) Q is the mid point of AB (iii) $PB = PA = \frac{1}{2} AC$.

Q.18 In figure AD and BE are two medians of ΔABC and BE$\parallel$DF. Prove that $CF = \frac{1}{4} AC$



Q.19 In ΔABC, AD is the median through A and E is the mid-point of AD, BE produced meets AC in F. Prove that $AF = \frac{1}{3} AC$.

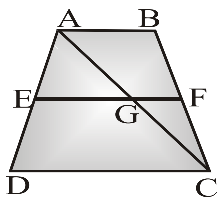


Q.20 P is the mid-point of side AB of a parallelogram ABCD. A line through B parallel to PD meets DC at Q and AD produced at R.

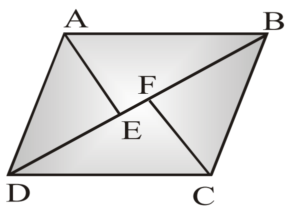
Prove that (i) AR = 2BC (ii) BR = 2BQ.

Q.21 In figure, ABCD is a trapezium in which side AB is parallel to side DC and E is the mid-points of side AD. If F is a point on the side BC such that the segments EF is parallel to side DC. Prove that F is the mid-point of BC and

$$EF = \frac{1}{2}(AB + DC)$$

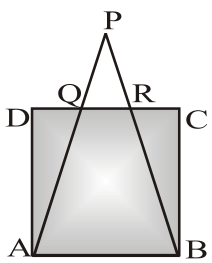


Q.22 In the given figure, ABCD is a parallelogram such that $\Delta AEB \cong \Delta CFD$.



Prove that $\angle DAE = \angle BCF$.

Q.23 In the figure, ABCD is a square and PAB is a triangle such that AQ = BR.



Prove that PQR is an isosceles triangle.

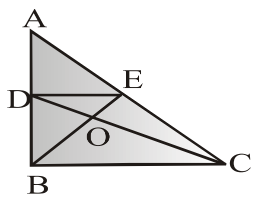
Q.24 In a ΔABC, D, E, and F are, respectively, the mid-points of BC, CA and AB. If the lengths of side AB, BC and CA are 7 cm, 8 cm and 9 cm, respectively, find the perimeter of ΔDEF.

Q.25 In a triangle ABC, $\angle A = 50°$, $\angle B = 60°$ and $\angle C = 70°$. Find the measures of the angles of the triangle formed by joining the mid-points of the sides of this triangle.

Q.26 In a triangle, P,Q, and R are the mid-points of sides BC, CA and AB respectively. If AC = 21 cm, BC = 29 cm and AB = 30 cm, find the perimeter of the quadrilateral ARPQ.

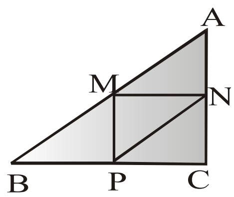
Q.27 In a ΔABC median AD is produced to X such that AD = DX. Prove that ABXC is a parallelogram.

Q.28 In figure, triangle ABC is right-angled at B. Given that AB = 9cm, AC = 15cm and D, E are the mid-points of the sides AB and AC respectively, calculate



(i) The length of BC (ii) The area of ΔADE

Q.29 In figure, M, N and P are the mid-points of AB, AC and BC respectively.

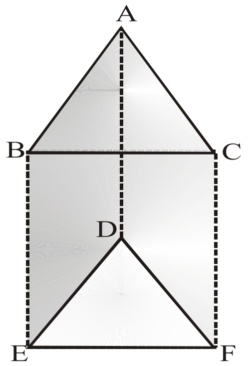


If MN = 3cm, NP = 3.5 cm and MP = 2.5cm, calculate BC, AB and AC.

Q.30 ABC is a triangle and through A, B, C lines are drawn parallel to BC, CA and AB respectively intersecting at P, Q and R. Prove that the perimeter of ΔPQR is double the perimeter of ΔABC.

Q.31 In a parallelogram, show that the angle bisector of two adjacent angles intersect at right angles.

Q.32 In figure ΔABC and ΔDEF are such that $AB = DE$, $BC = EF$, $AB \parallel DE$ and $BC \parallel EF$.



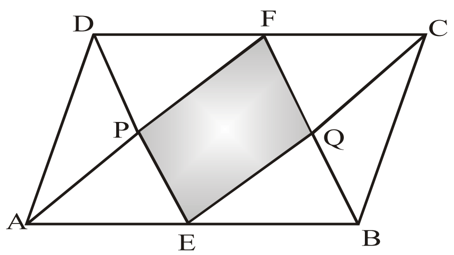
Show that

(i) ABED is a parallelogram.

(ii) BEFC is a parallelogram.

(iii) ADFC is a parallelogram.

Q.33 In figure, ABCD is a parallelogram. E and F are mid-points of the sides AB and CD respectively. AF and DE intersects at P ; BF and CE intersects at Q. **[NCERT]**



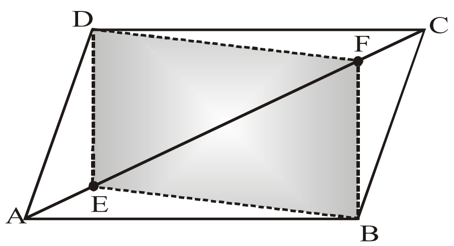
Prove that

(i) AECF is a parallelogram

(ii) BEDF is a parallelogram.

(iii) PEQF is a parallelogram.

Q.34 In figure, ABCD is a parallelogram. E and F are two point on the diagonal AC such that AE = CF.



Show that

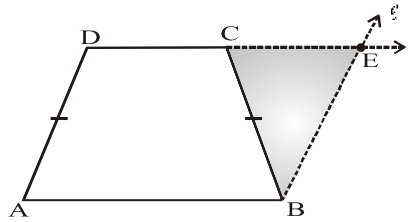
(i) $\Delta AEB \cong \Delta CFD$

(ii) $\Delta AED \cong \Delta CFB$

(iii) $\Delta BEF \cong \Delta DFE$

(iv) BEDF is a parallelogram.

Q.35 In figure, ABCD is a trapezium such that $AB \parallel CD$ and $AD = BC$. Line ℓ drawn through the vertex B and parallel to AD meets DC (produced) at E



Show that

(i) ABED is a parallelogram.

(ii) $\angle A + \angle C = \angle B + \angle D = 180°$.

ANSWER KEY

1. 37°, 143°, 37°, 143°

2. 108°, 72°, 108°, 72°

3. 68°, 112°, 68°, 112°

4. 4.5 cm

5. $\angle A = 45°$, $\angle B = 135°$

6. $\angle B = 110°$, $\angle C = 70°$, $\angle D = 110°$

7. $\angle CDB = 45°$, $\angle ADB = 60°$

9. $\angle C + \angle D = 180°$

10. $\angle A = \angle C = 45°$, $\angle B = \angle D = 135°$

11. $\angle AOB = 90°$

12. $\angle DBC = 50°$

24. 12 cm

25. 50°, 60°, 70°

26. 51 cm

28. 12 cm, 13.5 cm^2

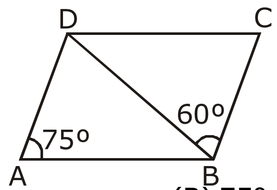
29. 6 cm, 7 cm, 5 cm

EXERCISE – III   MULTIPLE CHOICE QUESTIONS

Q.1 Three angles of a quadrilateral are 80º, 95º and 112º. Its fourth angles is
(A) 78º (B) 73º
(C) 85º (D) 100º

Q.2 The angles of a quarilateral are in the ratio 3 : 4 : 5 : 6. The smallest of these angles is
(A) 45º (B) 60º
(C) 36º (D) 48º

Q.3 In the given figure, ABCD is a parallelogram in which $\angle BAD = 75^o$ and $\angle CBD = 60^o$. Then, $\angle BDC$ =



(A) 60º (B) 75º
(C) 45º (D) 50º

Q.4 In which of the following figures are the diagonals
equal ?
(A) Parallelogram (B) Rhombus
(C) Trapezium (D) Rectangle

Q.5 If the diagonals of a quadrilateral bisect each other
at right angles, then the figure is a
(A) trapezium (B) parallelogram
(C) rectangle (D) rhombus

Q.6 The lengths of the diagonals of a rhombus are 16 cm and 12 cm. The length of each side of the rhombus is
(A) 10 cm (B) 12 cm
(C) 9 cm (D) 8 cm

Q.7 The length of each of side a rhombus is 10 cm and one of its diagonals is of length 16 cm. The length of the other diagonals is
(A) 13 cm (B) 12 cm
(C) $2\sqrt{39}$ cm (D) 6 cm

Q.8 If ABCD is a parallelogram with two adjacent angles $\angle A = \angle B$, then the parallelogram is a
(A) rhombus (B) trapezium
(C) rectangle (D) none of these

Q.9 In a quadrilateral ABCD , if AO and BO are the bisectors of $\angle A$ and $\angle B$ respectively, $\angle C = 70^o$ and $\angle D = 30^o$. Then $\angle AOB$ = ?
(A) 40º (B) 50º
(C) 80º (D) 100º

Q.10 The bisectors of any two adjacent angles of a parallelogram intersect at
(A) 30º (B) 45º
(C) 60º (D) 90º

Q.11 The biscetors of the angles of a parallelogram enclose a
(A) rhombus (B) square
(C) rectangle (D) parallelogram

Q.12 The figure formed by joining the mid-points of the adjacent sides of a quadrilateral is a
(A) rhombus (B) square
(C) rectangle (D) parallelogram

Q.13 The figure formed by joining the mid-points of the adjacent sides of a square is a
(A) rhombus (B) square
(C) rectangle (D) parallelogram

Q.14 The figure formed by joining the mid-points of the adjacent sides of a parallelogram is a
(A) rhombus (B) square
(C) rectangle (D) parallelogram

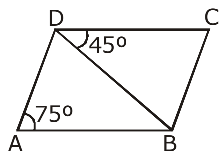
Q.15 The figure formed by joining the mid-points of the adjacent sides of a rectangle is a
(A) rhombus (B) square
(C) rectangle (D) parallelogram

Q.16 The figure formed by joining the mid-points of the adjacent sides of a rhombus is a
(A) rhombus (B) square
(C) rectangle (D) parallelogram

Q.17 If an angle of a parallelogram is two-third of its adjacent angle, the smallest angle of the parallelogram is
(A)108º (B) 54º
(C) 72º (D) 81º

Q.18 If one angle of a parallelogram is 24º less than twice the smallest angle, then the largest angle of the parallelogram is
(A) 68º (B) 102º
(C) 112º (D) 136º

Q.19 In the given figure, ABCD is a parallelogram in which $\angle BDC = 45^o$ and $\angle BAD = 75^o$. Then $\angle CBD$ =



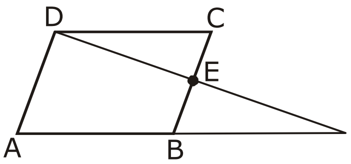
(A) 45º (B) 55º
(C) 60º (D) 75º

Q.20 If area of a ||gm with sides a and b is A and that of a rectangle with sides a and b is B, then

(A) A > B (B) A = B

(C) A < B (D) A $\geq$ B

Q.21 In the given figure ABCD is a ||gm and E is the mid-point of BC. Also, DE and AB when produced meet at F. Then :



(A) AF = $\frac{3}{2}$AB (B) AF = 2AB

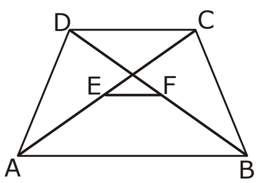
(C) AF = 3AB (D) $AF^2 = 2AB^2$

Q.22 The parallel sides of a trapezium are a and b respectively. The line joining the mid-points of its non-parallel sides will be

(A) $\frac{1}{2}(a-b)$ (B) $\frac{1}{2}(a+b)$

(C) $\frac{2ab}{(a+b)}$ (D) $\sqrt{ab}$

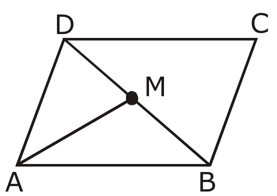
Q.23 In a trapezium ABCD, if E and F be the mid-points of the diagonals AC and BD respectively. Then EF =



(A) $\frac{1}{2}$AB (B) $\frac{1}{2}$CD

(C) $\frac{1}{2}$(AB + CD) (D) $\frac{1}{2}$(AB – CD)

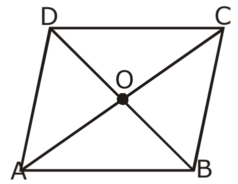
Q.24 In the given figure, ABCD is a parallelogram, M is the mid-point of BD and BD bisects $\angle B$ as well as $\angle D$. Then, $\angle AMB$ = ?



(A) 45º (B) 60º

(C) 90º (D) 30º

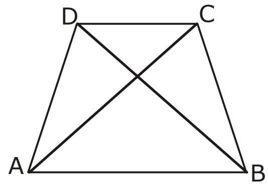
Q.25 In the given figure, ABCD is a rhombus. Then,



(A) $AC^2 + BD^2 = AB^2$ (B) $AC^2 + BD^2 = 2AB^2$

(C) $AC^2 + BD^2 = 4AB^2$ (D) $2(AC^2 + BD^2) = 3AB^2$

Q.26 In a trapezium ABCD, if AB||CD, then $(AC^2 + BD^2)$ =



(A) $BC^2 + AD^2 + 2BC.AD$

(B) $AB^2 + CD^2 + 2AB.CD$

(C) $AB^2 + CD^2 + 2AD.BC$

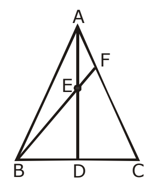
(D) $BC^2 + AD^2 + 2AB.CD$

Q.27 Two parallelogram stand on equal bases and between the same parallels. The ratio of their areas is

(A) 1 : 2 (B) 2 : 1

(C) 1 : 3 (D) 1 : 1

Q.28 In the given figure, AD is median of ΔABC and E is the mid-point of AD. If BE is joined and produced to meet AC in F, then AF = ?



(A) $\frac{1}{2}$AC (B) $\frac{1}{3}$AC

(C) $\frac{2}{3}$AC (D) $\frac{3}{4}$AC

Q.29 If $\angle A, \angle B, \angle C$ and $\angle D$ of a quadrilateral ABCD taken in order, are in the ratio 3 : 7 : 6 : 4, then ABCD is a

(A) rhombus (B) kite

(C) trapezium (D) parallelogram

Q.30 Which of the following is not true for a parallelogram

(A) Opposite sides are equal

(B) Opposite angles are equal

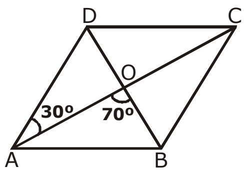
(C) Opposite angles are bisected by the diagonals

(D) Diagonals bisect each other

Q.31 If APB and CQD are two parallel lines,then the bisectors of $\angle APQ$, $\angle BPQ$, $\angle CQP$ and $\angle PQD$ enclose a

(A) square (B) rhombus
(C) rectangle (D) kite

Q.32 The diagonals AC and BD of a parallelogram ABCD intersect each other at the point O such that $\angle DAC = 30^o$ and $\angle AOB = 70^o$. Then, $\angle DBC$ =



(A) 40° (B) 35°
(C) 45° (D) 50°

Q.33 Three statements are given below :

I. In a ||gm, the angle biscetors of two adjacent angles enclose a right angle.

II. The angle bisectors of a ||gm form a rectangle.

III.The triangle formed by joining the mid-points of the sides of an isosceles triangle is not necessarily an isosceles triangle. Which is true

(A) I only (B) II only
(C) I and II (D) II and III

Q.34 Three statements are given below :

I. In a rectangle ABCD, the diagonal AC bisects $\angle A$ as well as $\angle C$.

II. In a square ABCD, the diagonal AC bisects $\angle A$ as well as $\angle C$.

III. In a rhombus ABCD, the diagonal AC bisects $\angle A$ as well as $\angle C$. Which is true ?

(A) I only (B) II and III
(C) I and III (D) I and II

Q.35 In which of the following is the lengths of diagonals equal ?

(A) Rhombus (B) Parallelogram
(C) Trapezium (D) Rectangle

Q.36 If the diagonals of a quadrilateral bisect each other at right angles, then it is a :

(A) Trapezium (B) Parallelogram
(C) Rectangle (D) Rhombus

Q.37 The length of the diagonals of a rhombus are 16 cm and 12 cm. The side of the rhombus is

(A) 10 cm (B) 12 cm
(C) 9 cm (D) 8 cm

Q.38 The length of a side of a rhombus is 5 m and one of its diagonals is of length 8 m. The length of the other diagonal is -

(A) 5 m (B) 7 m
(C) 6 m (D) 8 m

Q.39 If ABCD is a parallelogram with two adjacent angles A and B equal to each other, then the parallelogram is a :

(A) Rhombus (B) Trapezium
(C) Rectangle (D) None

Q.40 The bisectors of any two adjacent angles of a parallelogram intersect at -

(A) 30° (B) 45°
(C) 60° (D) 90°

Q.41 The bisectors of the angle of a || gm enclose a:

(A) parallelogram (B) rhombus
(C) rectangle (D) square

Q.42 The figure formed by joining the mid points of the adjacent sides of a quadrilateral is a:

(A) parallelogram (B) rectangle
(C) square (D) rhombus

Q.43 The figure formed by joining the mid points of the adjacent sides of a rectangle is a-

(A) square (B) rhombus
(C) trapezium (D) none

Q.44 The figure formed by joining the mid points of the adjacent sides of a rhombus is a-

(A) square (B) rectangle
(C) trapezium (D) none

Q.45 The figure formed by joining the mid points of the adjacent sides of a square is a :

(A) rhombus (B) square

(C) rectangle (D) parallelogram

Q.46 If one angle of a parallelogram is 24°less than twice the smallest angle, then the largest angle of the parallelogram is:

(A) 176° (B) 60°

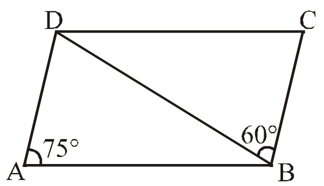
(C) 112° (D) 102°

Q.47 If an angle of a parallelogram is two-third of its adjacent angle, the smallest angle of the parallelogram is -

(A) 108° (B) 54°

(C) 72° (D) 81°

Q.48 In the given figure, ABCD is a parallelogram in which $\angle DAB = 75°$ and $\angle DBC = 60°$ Then, $\angle BDC$ is equal to :



(A) 75° (B) 60°

(C) 45° (D) 55°

Q.49 Three angles of a quadrilateral are of magnitudes 80°, 95° and 120°. The magnitude of the fourth angle is -

(A) 80° (B) 65°

(C) 75° (D) 70°

Q.50 If ABCD is a parallelogram and E, F are the centroids of Δ s ABD and BCD respectively, then EF equals -

(A) AE (B) BE

(C) CE (D) DE

Q.51 Two parallelograms stand on equal bases and between the same parallels. The ratio of their areas is -

(A) 1 : 2 (B) 2 : 1

(C) 1 : 1 (D) 1 : 3

Q.52 If a rectangle and a parallelogram are equal in area and have the same base and are situated on the same side, then the quotient: Perimeter of rectangle Perimeter of || gm is :

(A) equal to 1 (B) greater than 1

(C) less than 1 (D) indeterminate

Q.53 If ABCD is a rectangle, E, F are the mid points of BC and AD respectively and G is any point on EF, then Δ GAB equals :

(A) $\frac{1}{2}$(||ABCD) (B) $\frac{1}{3}$(||ABCD)

(C) $\frac{1}{4}$(||ABCD) (D) $\frac{1}{6}$(||ABCD)

Q.54 ABCD is a parallelogram, E, F are the mid points of BC and AD respectively and G is any point on EF. Then, area of ΔGAB equals-

(A) $\frac{1}{3}$(|| gm ABCD)

(B) $\frac{1}{4}$(|| gm ABCD)

(C) $\frac{1}{2}$(|| gm ABCD)

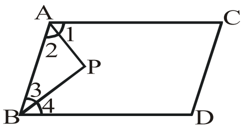
(D) $\frac{1}{6}$(|| gm ABCD)

ANSWER KEY

1.	B	**2.**	B	**3.**	C	**4.**	D
5.	D	**6.**	A	**7.**	B	**8.**	C
9.	B	**10.**	D	**11.**	C	**12.**	D
13.	B	**14.**	D	**15.**	A	**16.**	C
17.	C	**18.**	C	**19.**	C	**20.**	C
21.	B	**22.**	B	**23.**	D	**24.**	C
25.	C	**26.**	D	**27.**	D	**28.**	B
29.	C	**30.**	C	**31.**	C	**32.**	A
33.	C	**34.**	B	**35.**	D	**36.**	D
37.	A	**38.**	C	**39.**	C	**40.**	D
41.	C	**42.**	A	**43.**	B	**44.**	B
45.	B	**46.**	C	**47.**	C	**48.**	C
49.	B	**50.**	A	**51.**	C	**52.**	C
53.	C	**54.**	B				

EXERCISE – IV OLYMPIAD QUESTIONS

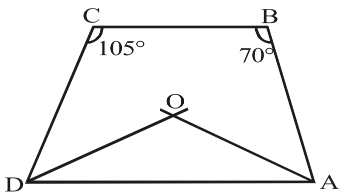
1. In the adjoining figure, AP and BP are angle bisectors of ∠A and ∠B which meet at P of the parallelogram ABCD. Then 2∠APB



(A) ∠A + ∠B (B) ∠A + ∠C

(C) ∠B + ∠D (D) ∠C + ∠D

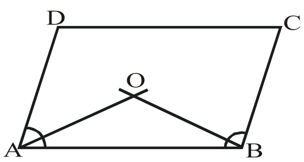
2. In the given figure, AO and DO are the bisector of the ∠A and the ∠D of the quadrilateral ABCD. Then the ∠AOD is



(A) 67.5° (B) 77.5°

(C) 87.5° (D) 99.75°

3. In a parallelogram the sum of the angle bisectors of two adjacent angles is



(A) 30° (B) 45°

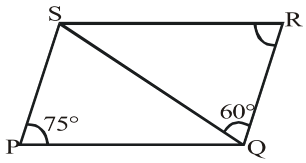
(C) 60° (D) 90°

4. In a parallelogram ABCD ∠D = 60° then the measurement of ∠A

(A) 120° (B) 65°

(C) 90° (D) 75°

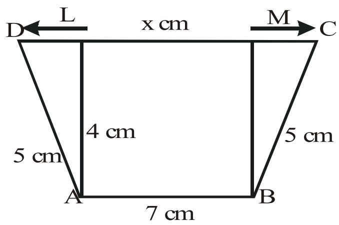
5. In the figure, parallelogram PQRS, the value of ∠SQP and ∠QSP are



(A) 45°, 60° (B) 60°, 45°

(C) 70°, 35° (D) 35°, 70°

6. In the given figure, ABCD is a trapezium in which AB = 7 cm, AD = BC = 5 cm, DC = x cm and the distance between AB and DC is 4 cm. Then the values of x is



(A) 13 cm (B) 16 cm

(C) 19 cm (D) cannot be determined

7. ABCD is a quadrilateral. If AC and BD bisect each other then ABCD must be

(A) Aquare (B) Rectangle

(C) Parallelogram (D) Rhomubs

8. ABCD is a parallelogram. The angle bisectors of ∠A and ∠D meet at O. The measure of ∠AOD is

(A) 45° (B) 90°

(C) Dependent on the angle A and D

(D) Cannot be determined from given data

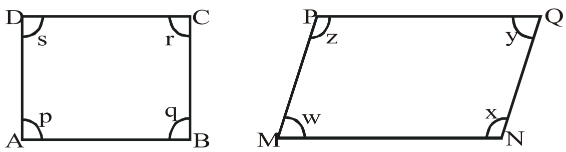
9. The diameter of circumcircle of a rectangle is 10

cm and breadth of the rectangle is 6 cm. Its length is

(A) 6 cm (B) 5 cm

(C) 8 cm (D) None of these

10. ABCD and MNOP are quadrilaterals as shown in the figure below. Then



(A) p + q + r + s = w + x + y + z

(B) p + q + r + s < w + x + y + z

(C) p + q + r + s > w + x + y + z

(D) None of the foregoing

11. If ABCD is a parallelogram, then ∠A – ∠C

(A) 180° (B) 0°

(C) 360° (D) 90°

12. One of the diagonals of a rhombus is equal to a side of the rhombus. The angles of the rhombus are

(A) 60° and 80° (B) 60° and 120°

(C) 120° and 240° (D) 100° and 120°

13. The diagonals of a parallelogram ABCD intersect at O. If ∠BOC = 90° and ∠BDC = 50°, then ∠AOB is

(A) 10° (B) 40°

(C) 50° (D) 90°

14. ABCD is a rhombus. If ∠ACB = 40°, then ∠ADB is

(A) 40° (B) 45°

(C) 50° (D) 60°

15. In a quadrilateral PQRS, if ∠P = ∠R = 100° and ∠S = 75° then ∠Q =

(A) 50° (B) 85°

(C) 120° (D) 360°

16. If the lengths of two diagonals of a rhombus are 12 cm and 16 cm, then the length of each side of the rhombus is

(A) 10 cm (B) 14 cm

(C) cannot be determined

(D) none of these

17. In a quadrilateral the angles are in the ratio 3 : 4 : 5 : 6. Then the difference between the greatest and the smallest angle is

(A) 108° (B) 54°

(C) 180° (D) 360°

18. In a quadrilateral ABCD, ∠A + ∠C + 180° then ∠B + ∠D =

(A) 360° (B) 100°

(C) 180° (D) 80°

19. Which of the following statement(s) is / are false ?

(A) Each diagonal of a quadrilateral divides it into two triangles

(B) Each side of a quadrilateral is less than the sum of the remaining three sides

(C) A quadrilateral can utmost have three obtuse angles

(D) A quadrilateral has four diagonals

20. The angles of a quadrilateral are x°, x – 10°, x + 30° and 2x°. Find the greatest angle

(A) 136° (B) 180°

(C) 68° (D) None of these

21. In a parallelogram ABCD, if ∠A = 80° then ∠B =

(A) 80° (B) 180°

(C) 100° (D) Data is not sufficient

22. In a trapezium ABCD, AB || CD if ∠A = 50° then ∠D =

(A) 110° (B) 130°

(C) 70° (D) 310°

23. In a rhombus ABCD, ∠A = 60° and AB = 6 cm. Find the diagonal BD

(A) $2\sqrt{3}$ cm (B) 6 cm

(C) 12 cm (D) Insufficient data

24. In a square ABCD, its diagonals bisect at O. Then the triangle AOB is

(A) An equilateral triangle

(B) An isosceles but not right angled triangle

(C) A right angled but not an isosceles triangle

(D) An isosceles right angled triangle

25. Which of the following properties are not true for a parallelogram ?

(A) Its diagonals are equal

(B) Its diagonals are perpendicular to each other

(C) The diagonals divide the figure into four congruent triangles

(D) All the above

26. In a rhombus ABCD, the diagonals intersect at O. If AB = 10 cm, diagonal BD = 16 cm, then the length of the diagonal AC is ....

(A) 12 cm (B) 6 cm

(C) 16 cm (D) 8 cm

27. The perimeter of a parallelogram is 180 cm. One side exceeds the another by 10 cm. The adjacent sides of the parallelogram are ....

(A) 30 cm, 40 cm (B) 40 cm, 50 cm

(C) 50 cm, 60 cm (D) None of these

28. ABCD is a parallelogram in which $\angle$DAO = 40°, $\angle$BAO = 35° and $\angle$COD = 65° then $\angle$ODC = ...

(A) 80° (B) 105°

(C) 25° (D) None of these

29. ABCD is a parallelogram in which $\angle$DAB = 75° and $\angle$DBC = 60° then $\angle$CDB = ...

(A) 60° (B) 75°

(C) 45° (D) 135°

30. If ABCD is a parallelogram, then $\angle A - \angle C$ =

(A) 180° (B) 0°

(C) 360° (D) 90°

QUADRILATERALS — ANSWER KEY — OLYMPIAD EXERCISE # 4

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	D	C	D	A	A	A	C	B	C	A
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	B	B	B	C	B	A	B	C	D	A
Que.	21	22	23	24	25	26	27	28	29	30
Ans.	C	B	B	D	D	A	B	A	C	B

AREA

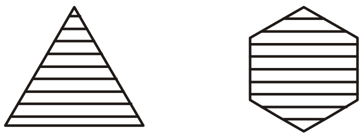
INTRODUCTION

In Previous classes, we have learn to find areas of plane figures, for example, area of triangle, rectangle, square, parallelogram, rhombus etc. In the present chapter, we shall study the relationship between the areas of these geometrical figures particularly when the two figures lie on same base and between same parallel lines.

Let us first understand the meaning of area of planar region and some axioms related to it.

Area of plane region. The part of a plane enclosed by a simple closed figure is called a planar region corresponding to that figure. The measure of this planar region in some unit is called the area of that planar region. Thus area of a figure is a number, associated with the part of the plane enclosed by the figure for example-

1. The part of the plane enclosed by a triangle is called the area of triangular region.

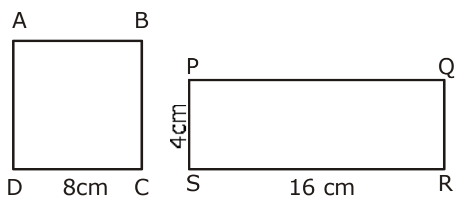


2. The part of the plane enclosed by a polygon is called area of the polygonal region.

Area axioms for plane figures. Following are the axioms related to area of plane figures.

1. Area Axiom of Congruent Figures.

We know that- "two plane figures are congruent' means they have same shape and size. If we place one figure on other, the two figures cover each other exactly. In other words, they have same area. Thus, we can say if R_1 and R_2 are two plane regions such that $R_1 \cong R_2$ then ar (R_1) = ar (R_2) e.g., if $\Delta ABC \cong \Delta PQR$ then ar (ΔABC) = ar (ΔPQR) if quad. ABCD $\cong$ quad. PQRS then ar (ABCD) = ar (PQRS). But converse of above is not true i.e., if areas of two plane regions are same then they need not be congruent. For example, a square ABCD of side 8 cm has area 64 cm^2 and a rectangle of sides 16 cm and 4 cm also has area 64cm^2 and a rectangle of sides 16 cm and 4 cm also has area 64 cm^2. But , clearly, square ABCD is not congruent to rectangle PQRS.



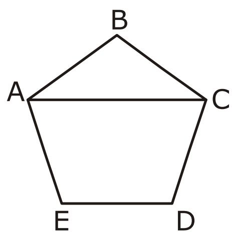
2. Axiom for Area of Union of Two Regions.

If R is a planar region, which is a union of two non- overlapping planar regions R_1 and R_2, then ar (R) = ar (R_1) + ar (R_2)

e.g., if R is a polygonal region ABCDE which is the union of two regions.

R_1 : the triangular region ABC.

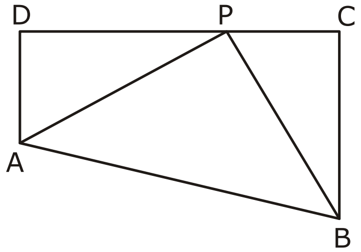
R_2 : the quadrilateral region CEDA. then,



ar (Polygon ABCDE) = ar (ΔABC) + ar (quad. CDEA).

3. Axiom for Area of Included Region.

If R_1 be a plane region included in any other planar region R_2, then ar $(R_1) \leq$ ar (R_2) e.g., in the adjacent figure, triangular region ABP is included inside the quadrilateral region ABCD, therefore



ar (ΔABP) $\leq$ ar (quad. ABCD).

4. Axiom for Area of a Rectangle.

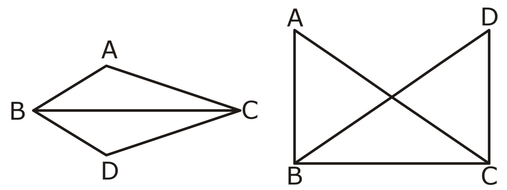
If a rectangle ABCD has length l and breadth b then ar (rect. ABCD) = l × b.

Using above axioms, we can derive the formulae for area of parallelogram, triangle, trapezium and rhombus. It also needs the study of relationship between the areas of these geometric figures under the condition when they lie on the same base and between the same parallels.

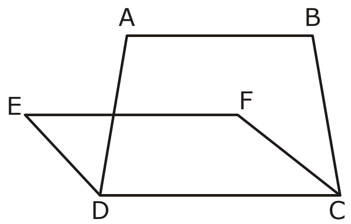
GEOMETRIC FIGURES ON THE SAME BASE AND BETWEEN THE SAME PARALLELS

Let us first understand the meaning of 'same base'. Two geometric figures are said to have same base if they have one side common. For example, in the following cases figures are on same base.

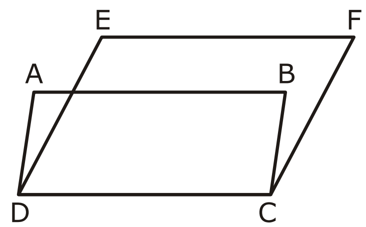
(i) ΔABC and ΔDBC are on the same base BC in each of the two figures given here.



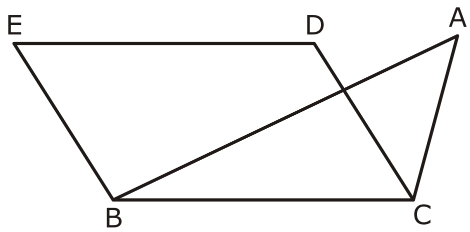
(ii) In the adjacent figure trapezium ABCD and parallelogram CDEF are on the same base CD.



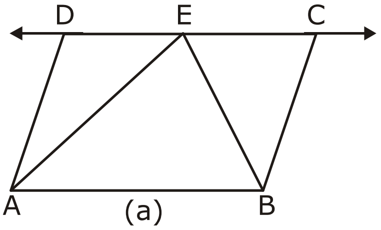
(iii) Two parallelograms ABCD and CDEF are on the same base CD in the figure given below-



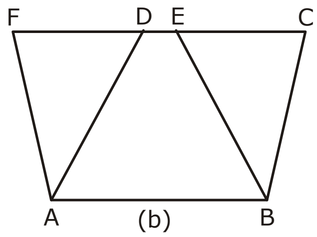
(iv) ΔABC and parallelogram BCDE lie on the same base BC as shown in the given figure.



Now two plane geometric figures are said to be on the same base and between the same parallels if each of these have one side common and their opposite sides or vertex lie along or on a line parallel to the base and on the same side of base. For example, in fig. (a) parallelogram ABCD and ΔABE lie on the same base AB and between the same parallels AB and DC.

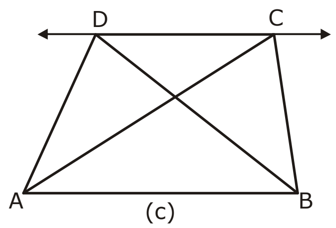


(a)



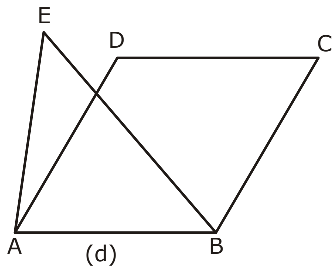
(b)

In fig. (b) parallelograms ABCD and ABEF lie on the same base AB and between the same parallels AB and FC.



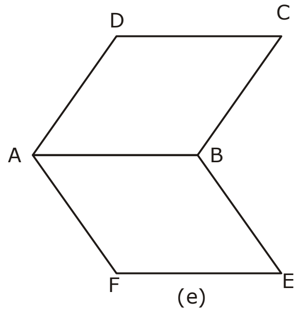
(c)

In fig. (c) triangles ABC adn ABD lie on the same base AB and between the same parallels AB and DC.



(d)

It should be noted that the figures (d) and (e) lie on the same but not between the same base parallels. In fig. (d) vertex E of ΔABE and opposite side of parallelogram ABCD do not lie on the same line parallel to base AB while in Fig. (e) parallelogram ABCD and parallelogram ABEF have their sides opposite to base on different sides of base and not on the same side of base.

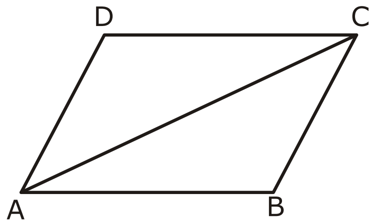


(e)

So figures (d) and (e) cannot be considered as the figures on same base and between the same parallels.

SOLVED PROBLEMS

Ex.1 Diagonal of a parallelogram divides it into two triangles of equal area.



Sol. **Given.** ABCD is a ||gm and AC is diagonal.

To prove : ar (ABC) = ar (ADC).

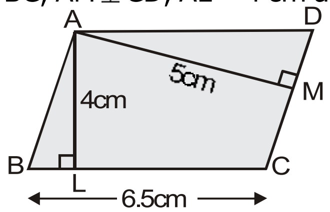
Proof. We know that diagonal of a parallelogram divides it into two congruent triangles.

$\because$ $\Delta ABC \cong \Delta CDA$

$\therefore$ ar (ΔABC) = ar (ΔCDA)

(by area axiom of congruent figures)

Ex.2 In fig, ABCD is a parallelogram, $AL \perp BC$, $AM \perp CD$, AL = 4 cm and AM = 5 cm. If BC = 6.5 cm, then find CD.

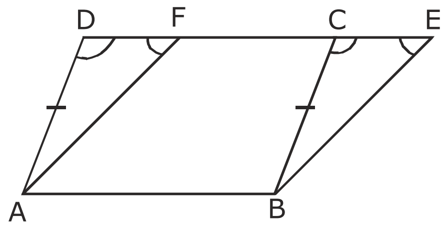


Sol. We have, BC × AL = CD × AM (Each equal to area of the parallelogram ABCD)

$\Rightarrow$ 6.5 × 4 = CD × 5

$\Rightarrow$ CD = $\frac{6.5 \times 4}{5}$ cm $\Rightarrow$ CD = 5.2 cm.

Ex.3 Parallelogram on the same base and between the same parallels are equal in area. **[NCERT]**
(CBSE 2010)



Sol. **Given.** Two parallelograms ABCD and ABEF on the same base AB and between the same parallels AB and DE.

To Prove : ar (||gm ABCD) = ar (||gm ABEF).

Proof. In ΔADF and ΔBCE.

$\because$ AD||BC being opposite sides of a parallelogram and DC a transversal,

$\therefore$ $\angle$ADF = $\angle$BCE (corresponding angles)

Also since AF ||BE being opposite sides of a parallelogram and DE a transversal,

$\therefore$ $\angle$AFD = $\angle$BEC (corresponding angles)

$\therefore$ $\angle$DAF = $\angle$CBE ($\because$ if two angles of two triangles are equal, third will also be equal)

and, AD = BC (opp. sides of a||gm)

$\therefore$ $\Delta ADF \cong \Delta BCE$ (ASA congruence condition)

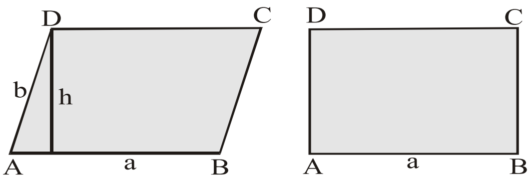
$\therefore$ ar (ΔADF) = ar (ΔBCE)

$\Rightarrow$ ar (ΔADF) + ar (||gm ABCF) = ar (ΔBCE)
+ ar (||gm ABCF)

$\Rightarrow$ ar (||gm ABCD) = ar (||gm ABEF)

Ex.4 Prove that of all parallelograms of which the sides are given, the parallelogram which is rectangle has the greatest area.

Sol. Let ABCD be a parallelogram in which AB = a and AD = b. Let h be the altitude corresponding to the base AB. Then,



ar (||gm ABCD) = AB × h = ah

Since the sides a and b are given. Therefore, with the same sides a and b we can construct infinitely many parallelograms with different heights.

Now, ar (||gm ABCD) = ah

$\Rightarrow$ ar (||gm ABCD) is maximum or greatest when h is maximum. [$\because$ a is given i.e., a is constant]

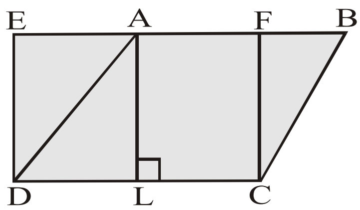
But, the maximum value which h can attain is AD = b and this is possible when AD is perpendicular to AB i.e. the ||gm ABCD becomes a rectangle.

Thus, (ar ||gm ABCD) is greatest when AD $\perp$ AB i.e. when (||gm ABCD) is a rectangle.

Ex.5 In fig, ABCD is a parallelogram and EFCD is a rectangle. Also AL $\perp$ DC. Prove that

(i) ar (ABCD) = (EFCD) (ii) ar (ABCD) = DC × AL

Sol. (i) We know that a rectangle is also a parallelogram.



Thus, parallelogram ABCD and rectangle EFCD are on the same base CD and between the same parallels CD and BE.

$\therefore$ ar (||gm ABCD) = ar (EFCD)

(ii) From (i), we have ar (ABCD) = ar (EFCD)

$\Rightarrow$ ar (ABCD) = CD × FC

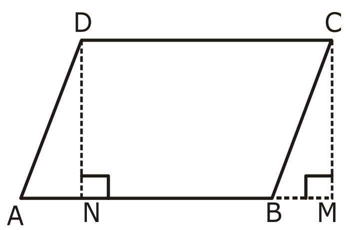
[$\because$ Area of a rectangle = Base × Height]

$\Rightarrow$ ar (ABCD) = CD × AL

[$\because$ AL = FC as ALCF is a rectangle]

$\Rightarrow$ ar (ABCD) = DC × AL

Ex.6 Area of a parallelogram is equal to the product of its base and corresponding altitude. **[NCERT]**



Sol. **Given.** A parallelogram ABCD in which DN is altitude corresponding to base AB.

To Prove : ar (||gm ABCD) = AB × DN.

Construction. Draw CM perpendicular to AB which meets AB produced at M.

Proof. DN and CM are both perpendicular to same line AB,

$\therefore$ DN ||CM

Also DN = CM

($\because$ each is distance between two parallel lines AB and DC)

$\therefore$ DCMN is a parallelogram. ($\because$ A quad. is a parallelogram if one pair of opp. side is equal and parallel)

Also $\angle$DNM = 90º, So DCMN is a rectangle.

$\therefore$ ar (||gm DCMN) = DC × DN

($\because$ Area of rect. = length × breadth)

But, AB = DC

($\because$ opp. sides of a parallelogram are equal)

$\therefore$ ar (||gm DCMN) = AB × DN ...(1)

Also as both the parallelograms (DCMN and ABCD) lie on the same base DC and between the same parallels DC and AM.

$\therefore$ ar (|| gm DCMN) = ar (||gm ABCD)...(2)

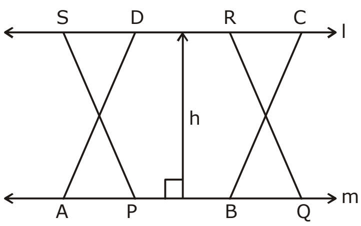
($\because$ parallelograms on same base and between same parallels are equal in area)

From equations (1) and (2), we get

ar (ABCD) = AB × DN = base

× corresponding altitude.

corollary. Parallelograms on equal base and between the same parallels are equal in area.



Proof. If ABCD adn PQRS be two parallelograms on equal base AB and PQ ann between same parallel lines l and m, and h be the perpendicular distance between l and m, then

ar (||gm ABCD) = AB × h ...(1)

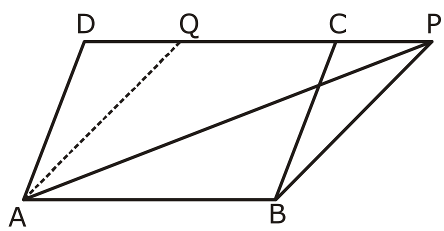
and ar (||gm PQRS) = PQ× h ...(2)

But AB = PQ (given) ...(3)

From equations (1), (2) and (3), we get

ar (ABCD) = ar (||gm PQRS)

Ex.7 If a triangle and a parallelogram lie on the same base and between the same parallel then area of triangle is equal to half of the area of parallelogram. **[NCERT]**



Sol. **Given.** ΔABP and a ||gm ABCD on same base AB and between the same parallels AB and DP.

To prove : ar (ΔABP) = $\frac{1}{2}$ar (||gm ABCD).

Construction. Through A , draw a line AQ parallel to BP intersecting DP at Q.

Proof. AB||DC ($\because$ opp. sides of a parallelogram are parallel).

$\therefore$ AB || QP

Also, AQ || BP (by construction)

$\therefore$ ABPQ is a parallelogram.

Thus ABCD and ABPQ are two parallelograms on the same base AB and between the same parallels AB and DP.

$\therefore$ ar (||gm ABCD) = are (||gm ABPQ) ...(1)

Also in parallelogram ABPQ, AP is the diagonal

$\therefore$ ar (Δ ABP) = ar (Δ AQP)

($\because$ diagonal of a parallelogram divides it into two triangles of equal area).

But ar (ABP) + ar (AQP) = ar (ABPQ)

$\Rightarrow$ ar (ABP) + ar (ABP) = ar (ABPQ)

$\Rightarrow$ 2ar (ABP) = ar (ABPQ) ...(2)

From equations (1) and (2), we get

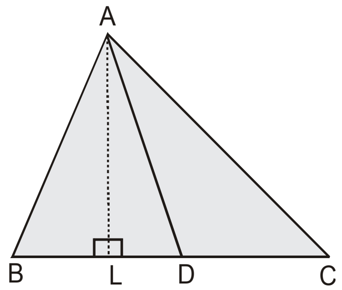
2ar (Δ ABP) = ar (||gm ABCD)

$\Rightarrow$ ar (Δ ABP) = $\frac{1}{2}$ ar (||gm ABCD)

Hence area of triangle is half the area of parallelogram.

Ex.8 Show that a median of a triangle divides it into two triangles of equal area. **[NCERT]**

Sol. **Given :** ΔABC in which AD is a median.



To prove : ar(ΔABD) = ar (ΔADC).

Construction : Draw AL $\perp$ BC.

Proof : Since AD is the median ΔABC.

Therefore, D is the mid-point of BC.

$\Rightarrow$ BD = DC

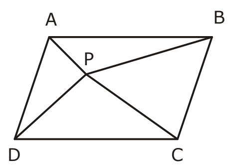
$\Rightarrow$ BD × AL = DC × AL [Multiplying both sides by AL]

$\Rightarrow$ $\frac{1}{2}$(BD × AL) = $\frac{1}{2}$(DC × AL) [Multiplying both sides by $\frac{1}{2}$]

$\Rightarrow$ ar (ΔABD) = ar (ΔADC)

Hence, proved.

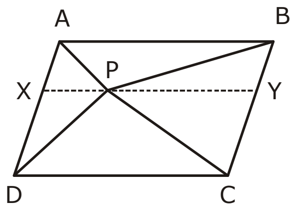
Ex.9 In the given figure, P is a point in the interior of a parallelogram ABCD. Show that **[NCERT]**



(i) ar (ΔAPB) + ar (Δ PCD) = $\frac{1}{2}$ ar (||gm ABCD)

(ii) ar (ΔAPD) + ar (ΔPBC) = ar (ΔAPB) ar (ΔPCD).

Sol. Let us draw a line through P parallel to AB which meet AD at X and BC at Y.



$\because$ AD || BC (opp. sides of a parallelogram)

$\therefore$ AX || BY

Also AB || XY (by construction)

$\therefore$ ABYX is a parallelogram.

Similarly CDXY is a parallelogram.

Now parallelogram ABYX and ΔAPB lie on the same base AB and between the same parallels AB and XY,

$\therefore$ ar (Δ APB) = $\frac{1}{2}$ ar (||gm ABYX) ...(1)

And parallelogram CDXY and ΔPDC lie on the same base DC and between the same parallels DC and XY,

$\therefore$ ar (Δ PCD) = $\frac{1}{2}$ ar (||gm CDYX) ...(2)

Adding equations (1) and (2), we get

ar (ΔAPB) + ar (Δ PCD) = $\frac{1}{2}$ ar (||gm ABYX) + $\frac{1}{2}$ ar (||gm CDXY)

ar (ΔAPB) + ar (Δ PCD) = $\frac{1}{2}$ (ar (||gm ABYX) + ar (||gm CDXY))

ar (ΔAPB) + ar (Δ PCD) = $\frac{1}{2}$ ar (||gm ABCD)

$\therefore$ ar (Δ APB) + ar (Δ PCD) = $\frac{1}{2}$ ar (||gm ABCD) ...(3)

Also, ar (ΔAPB) + ar (ΔPBC) + ar (ΔAPD) + ar (Δ PCD) = ar (||gm ABCD).

$\Rightarrow$ ar (ΔAPD) + ar (ΔPBC) + $\frac{1}{2}$ ar (||gm ABCD) = ar (||gm ABCD) (using (3))

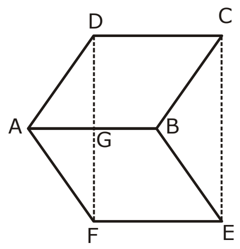
$\Rightarrow$ ar (ΔAPD) + ar (Δ PBC) = ar (||gmABCD) – $\frac{1}{2}$ ar(||gm ABCD)

$\Rightarrow$ ar (ΔAPD) + ar (ΔPBC) = $\frac{1}{2}$ ar (||gm ABCD) ...(4)

From equations (3) and (4), we get

ar (ΔAPD) + ar (ΔPBC) = ar (ΔAPB) + ar (ΔPCD).

Ex.10 ABCD and ABEF are two parallelograms on the opposite sides of AB as shown in the figure. CE and DF are joined. Prove that : (i) ar (Δ ADF) = ar (Δ BCE) (ii) ar (CDFE) = ar (||gm ABCD) + ar (||gm ABEF).



Sol. $\because$ ABCD is a parallelogram,

$\therefore$ AB||DC and AB = DC ...(1)

Also as ABEF is a parallelogram,

$\therefore$ AB||EF and AB = EF ...(2)

From equations (1) and (2), we get

DC||EF and DC = EF.

$\therefore$ DCEF is a parallelogram.

($\because$ one pair of opp. sides of a quad. are equal and parallel)

$\therefore$ DF = CE.

Now in Δ ADF and Δ BCE

AD = BC (opp. sides of a ||gm ABCD)

AF = BE (opp. sides of a ||gm ABEF)

DF = CE (proved earlier)

$\therefore$ ΔADF $\cong$ ΔBCE (SSS congruence condition)

$\therefore$ ar (ΔADF) = ar (ΔBCE) ...(3)

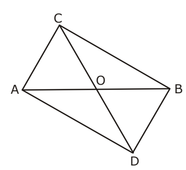
Again ar (||gm ABCD) + ar (||gn ABEF) = ar (ΔADF) + ar (||gm BCDG) + ar (||gm BGFE)

ar (||gm ABCD) + ar (||gn ABEF) = ar (ΔBCE) + ar (||gm BCDG) + ar (||gm BGFE)

ar (||gm ABCD) + ar (||gn ABEF) = = ar (||gm CDFE) (using eqn. (3)

Hence ar (||gm CDFE) = ar (||gm ABCD) + ar (||gm ABEF).

Ex.11 In the given figure ABC and ABD are two triangles on the same base AB. If line segment CD is bisected by AB at O, show that : ar (ΔABC) = ar (ΔABD).



Sol. Given that AB bisects CD, i.e., O is the mid point of CD. Now in ΔADC, AO is the median.

$\therefore$ ar (ΔACO) = ar (ΔADO) .....(1)

($\because$ median of a triangle divides it into two triangles of equal area)

Also in ΔBCD, BO is the median,

$\therefore$ ar (ΔBCO) = ar (ΔBDO) .....(2)

($\because$ median of a triangle divides it into two triangles of equal area)

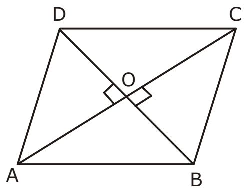
Adding equations (1) and (2) we get

ar (ΔACO) + ar (ΔBCO) = ar (ΔADO) + ar (ΔBDO)

$\Rightarrow$ ar (ΔABC) = ar (ΔABD).

Ex.12 Prove that area of a rhombus is equal to the half of the product of its diagonals.

Sol. Let ABCD be a rhombus. Let its diagonals AC and BD intersect each other at O.



We know that diagonals of a rhombus bisect each other at 90º.

$\therefore$ DO is altitude of ΔADC and BO is the altitude of ΔABC.

Now ar (ΔABC) = $\frac{1}{2} \times AC \times OB$ ...(1)

and ar (ΔADC) = $\frac{1}{2} \times AC \times OD$ ...(2)

($\because$ area of triangle = $\frac{1}{2} \times$ base$\times$altitude)

Adding equations (1) and (2) we get

ar (ΔABC) + ar (ΔADC) = $\frac{1}{2} \times AC \times OB + \frac{1}{2} \times AC \times OD$

$\Rightarrow$ ar (||gm ABCD) = $\frac{1}{2} \times AC\,(OB \times OD)$

$\Rightarrow$ ar (||gm ABCD) = $\frac{1}{2} \times AC \times BD$

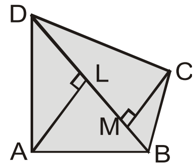
or area of rhombus = $\frac{1}{2} \times$ (product of diagonals).

EXERCISE – I   UNSOLVED PROBLEMS

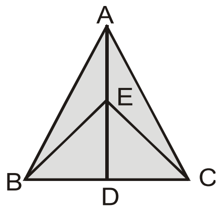
Q.1 ABCD is a parallelogram M is any point on diagonal BD. Prove that. ar (ΔABM) = ar (ΔBCM).

Q.2 ABCD is a parallelogram. Diagonals AC and BD intersect each other at O. A line through O is drawn which meets AB at P and DC at Q. Prove that : ar (OAQ) = ar(OCP).

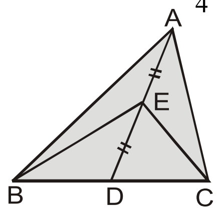
Q.3 ABCD is a quadrilateral. If AL $\perp$ BD and CM $\perp$ BD, prove that: ar (quad. ABCD) = $\frac{1}{2}$ × BD × (AL + CM).



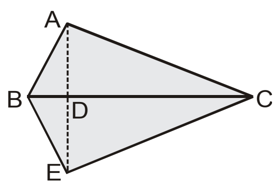
Q.4 In the given figure, D is the mid-point of BC and E is any point on AD. Prove that: **[NCERT]**
(i) ar (ΔEBD) = ar (ΔEDC), (ii) ar (ΔABE) = ar (ΔACE)



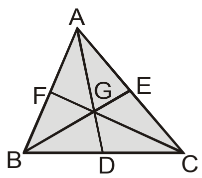
Q.5 In the given figure, D is the mid-point of BC and E is the, mid-point of AD. **[NCERT]**
Prove that : ar (ΔABE) = $\frac{1}{4}$ar (ΔABC).



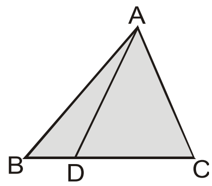
Q.6 In the given figure, a point D is taken on side BC of ΔABC and AD is produced to E, making DE = AD. Show that : ar (ΔBEC) = ar (ΔABC).



Q.7 If the medians of a ΔABC intersect at G, show that : ar (ΔAGB) = ar (ΔAGC) = ar (ΔBGC) = $\frac{1}{3}$ ar (ΔABC).



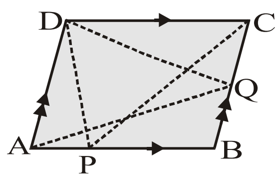
Q.8 D is a point on base BC of a ΔABC such that 2BD = DC. Prove that : ar (ΔABD) = $\frac{1}{3}$ ar (ΔABC).



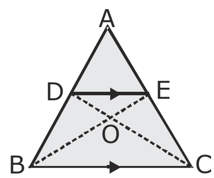
Q.9 D, E, F are respectively the mid points of sides BC, CA and AB of ΔABC. O is any point on AD. Prove that : ar(ΔBOF) = ar(ΔCOE)

Q.10 The diagonals AC and BD of a parallelogram ABCD intersect each other at a point O. Through O, a line is drawn to intersect AD and BC at points x and y respectively. Show that xy divides the parallelogram into two parts of equal area.

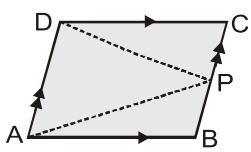
Q.11 In the adjoining figure, ABCD is a parallelogram. P and Q are any two points on the sides AB and BC respectively.
Prove that : ar (ΔCPD) = ar (ΔAQD)



Q.12 In the adjoining figure, DE || BC. Prove that: (i) ar (ΔABE) = ar (ΔACD). (ii) ar (ΔOBD) = ar (ΔOCE).



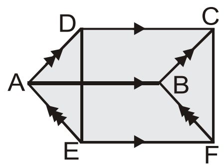
Q.13 In the given figure, ABCD is a parallelogram and P is a point on BC. Prove that : ar (ΔABP) + ar (ΔDPC) = ar (ΔAPD).



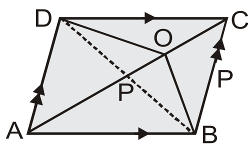
Q.14 ABCD is a quadrilateral. A line through D, parallel to AC meets BC produced in P. Prove that area of ΔABP = area of quadrilateral ABCD.

Q.15 Show that the area of rhombus is half the product of the lengths of its diagonals.

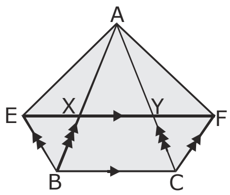
Q.16 In the adjoining figure, two parallelograms ABCD and AEFB are drawn on opposite sides of AB. Prove that: ar (|| gm ABCD) + ar (|| gm AEFB) = ar (|| gm EFCD).



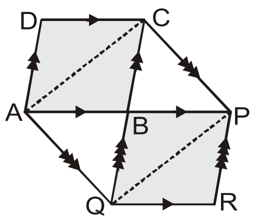
Q.17 In the adjoining figure, ABCD is a parallelogram and O is any point on its diagonal AC. Show that : ar (ΔAOB) = ar (ΔAOD).



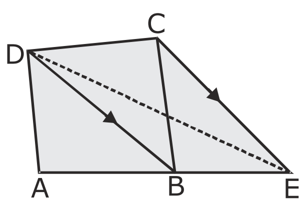
Q.18 In the given figure, XY||BC, BE||CA and FC||AB. Prove that : ar (ΔABE) = ar (ΔACF).



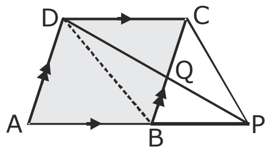
Q.19 In the given figure, the side AB of ||gm ABCD is produced to a point P. A line through A drawn parallel to CP meets CB produced in Q and the parallelogram PBQR is completed. Prove that: ar (|| gm ABCD) = ar (|| gm BPRQ).



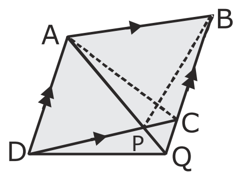
Q.20 In the adjoining figure, CE is drawn parallel to DB to meet AB produced at E. Prove that: ar (quad. ABCD) = ar (ΔDAE).



Q.21 In the adjoining figure, ABCD is a parallelogram. AB is produced to a point P and DP intersects BC at Q. Prove that: ar (ΔAPD) = ar (quad. BPCD).



Q.22 In the adjoining figure, ABCD is a parallelogram. Any line through A cuts DC at a point P and BC produced at Q. Prove that: ar (ΔBPC) = ar (ΔDPQ).



Q.23 P is any point on diagonal BD of parallelogram ABCD, prove that ar(ΔABP) = ar(ΔBCP).

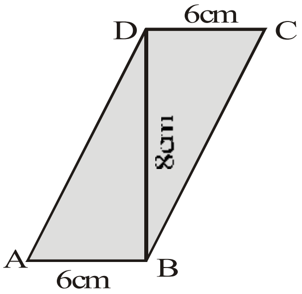
Q.24 Median AD of ΔABC is produced till E such that AD = DE. Prove that AC || BE.

Q.25 ABCD is a parallelogram. P and Q are any points on sides AB and CD respectively. AQ and DP intersect each other at X while PC and BQ intersect each other at Y. Prove that :

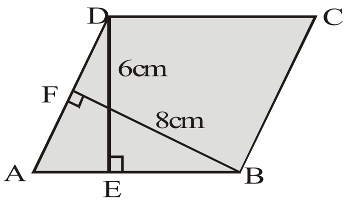
ar(ΔAXD) = ar(ΔBYC).

EXERCISE – II | SCHOOL EXAM/BOARD

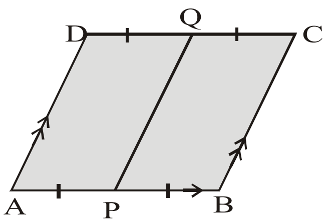
Q.1 In the adjoining figure, BD is a diagonal of quad. ABCD. Show that ABCD is a parallelogram and calculate the area of || gm ABCD.



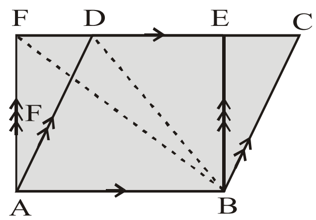
Q.2 In a || gm ABCD, it is given that AB = 16 cm and the altitudes corresponding to the sides AB and AD are 6 cm and 8 cm respectively. Find the length of AD.



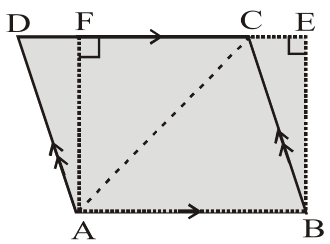
Q.3 Show that the line segment joining the mid-points of a pair of opposite sides of a parallelogram, divides it into two equal parallelograms.



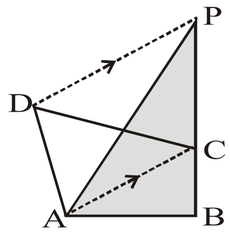
Q.4 In the given figure, the area of || gm ABCD is 90 cm^2. State giving reasons :
(i) ar (||gm ABEF) (ii) ar (ΔABD) (iii) ar (ΔBEF).



Q.5 In the given figure, the area of ΔABC is 64 cm^2. State giving reasons :
(i) ar (|| gm ABCD) (ii) ar (rect. ABEF)



Q.6 In the given figure, ABCD is a quadrilateral. A line through D, parallel to AC, meets BC produced in P. Prove that : ar (ΔABP) = ar (quad. ABCD).



Q.7 Answer the following questions as per the exact requirement:

(i) ABCD is a parallelogram in which AB||CD and AB = CD = 10 cm. If the perpendicular distance between AB and CD be 8 cm, find the area of the parallelogram ABCD.

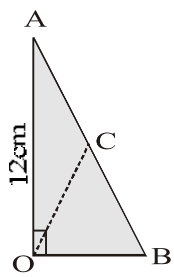
(iI) ABCD is a parallelogram having area 240 cm^2, BC = AD = 20 cm and BC||AD. Find the distance between the parallel sides BC and AD.

(iii) ABCD is a parallelogram having area 160 cm^2, BC||AD and the perpendicular distance between BC and AD is 10 cm. Find the length of the side BC.

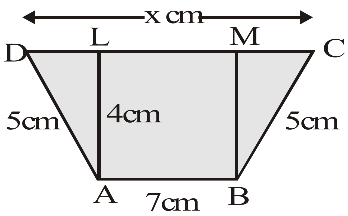
(iv) ABCD is a parallelogram having area 200 cm^2. If AB||CD, P is mid-point of AB and Q is mid-point of CD, find the area of the quadrilateral APQD.

(v) ABCD is a parallelogram having area 450 cm^2. If AB||CD, points P and Q divide AB and DC respectively in the ratio 1 : 2, find the area of the parallelogram APQD and parallelogram PBCQ.

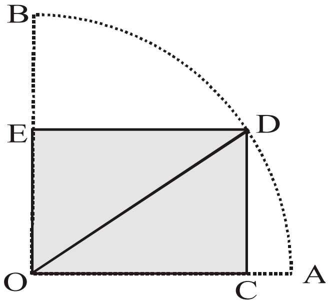
Q.8 In fig, $\angle$AOB = 90°, AC = BC, OA = 12 cm and OC = 6.5 cm. Find the area of ΔAOB.



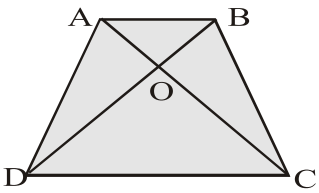
Q.9 In fig, ABCD is a trapezium in which AB = 7 cm, AD = BC = 5 cm, DC = x cm, and distance between AB and DC is 4 cm. Find the value of x and area of trapezium ABCD.



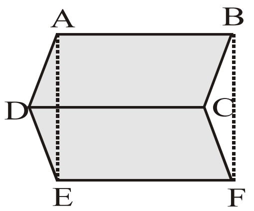
Q.10 In fig, OCDE is a rectangle inscribed in a quadrant of a circle of radius 10 cm. If OE $= 2\sqrt{5}$, find the area of the rectangle.



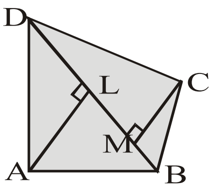
Q.11 In fig, ABCD is a trapezium in which AB∥DC. Prove that ar(ΔAOD) = ar(ΔBOC).



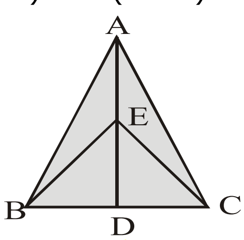
Q.12 In fig, ABCD, ABFE and CDEF are parallelograms. Prove that ar(ΔADE) = ar(ΔBCF).



Q.13 ABCD is a quadrilateral. If AL ⊥ BD and CM ⊥ BD, prove that: ar (quad. ABCD) = $\frac{1}{2}$ × BD × (AL + CM).

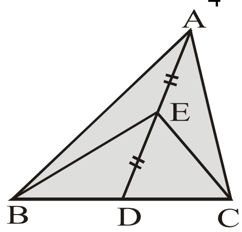


Q.14 In the given figure, D is the mid-point of BC and E is any point on AD. Prove that:
(i) ar (ΔEBD) = ar (ΔEDC).
(ii) ar (ΔABE) = ar (ΔACE).

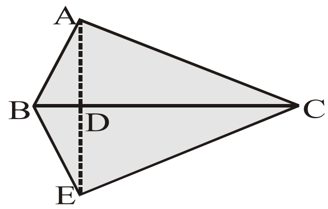


Q.15 In the given figure, D is the mid-point of BC and E is the, mid-point of AD.

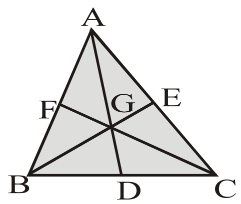
Prove that : ar (ΔABE) = $\frac{1}{4}$ ar (ΔABC).



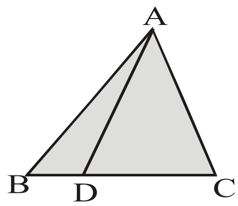
Q.16 In the given figure, a point D is taken on side BC of ΔABC and AD is produced to E, making DE = AD. Show that : ar (ΔBEC) = ar (ΔABC).



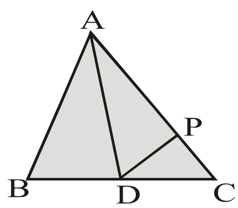
Q.17 If the medians of a ΔABC intersect at G, show that : ar (ΔAGB) = ar (ΔAGC) = ar (ΔBGC) = $\frac{1}{3}$ ar (ΔABC).



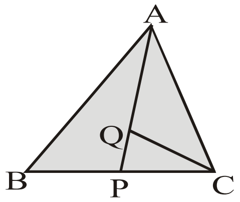
Q.18 D is a point on base BC of a ΔABC such that 2BD = DC. Prove that : ar (ΔABD) = $\frac{1}{3}$ ar (ΔABC).



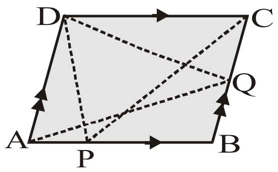
Q.19 In the given figure, AD is a median of ΔABC and P is a point on AC such that :
ar (ΔADP) : ar (ΔABD) = 2 : 3.
Find : (i) AP : PC (ii) ar (ΔPDC) : ar (ΔABC).



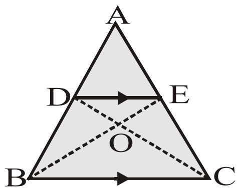
Q.20 In the given figure, P is a point on side BC of ΔABC such that BP : PC = 1 : 2 and Q is a point on AP such that PQ : QA = 2 : 3. Show that ar (ΔAQC) : ar (ΔABC) = 2 : 5.



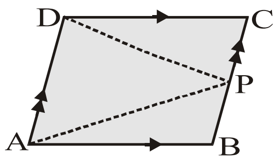
Q.21 In the adjoining figure, ABCD is a parallelogram. P and Q are any two points on the sides AB and BC respectively.
Prove that : ar (ΔCPD) = ar (ΔAQD)



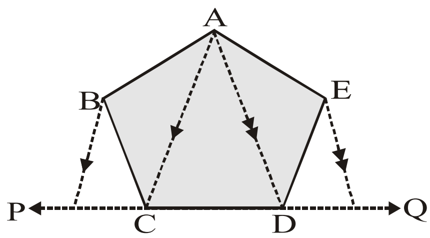
Q.22 In the adjoining figure, DE II BC. Prove that: (i) ar (ΔABE) = ar (ΔACD). (ii) ar (ΔOBD) = ar (ΔOCE).



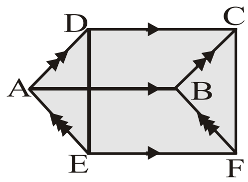
Q.23 In the given figure, ABCD is a parallelogram and P is a point on BC. Prove that : ar (ΔABP) + ar (ΔDPC) = ar (ΔAPD).



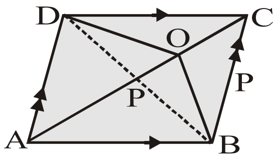
Q.24 In the adjoining figure, ABCDE is a pentagon. BP drawn parallel to AC meets DC produced at P and EQ drawn parallel to AD meets CD produced at Q. Prove that : ar (Pentagon ABCDE) = ar (ΔAPQ).



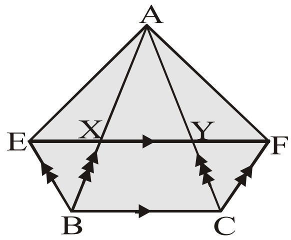
Q.25 In the adjoining figure, two parallelograms ABCD and AEFB are drawn on opposite sides of AB.
Prove that: ar (|| gm ABCD) + ar (|| gm AEFB) = ar (|| gm EFCD).



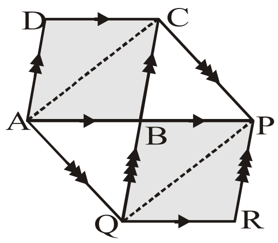
Q.26 In the adjoining figure, ABCD is a parallelogram and O is any point on its diagonal AC.
Show that : ar (ΔAOB) = ar (ΔAOD).



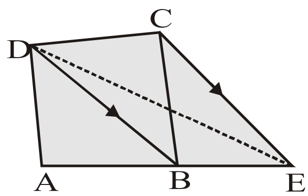
Q.27 In the given figure, XY||BC, BE||CA and FC||AB. Prove that : ar (ΔABE) = ar (ΔACF).



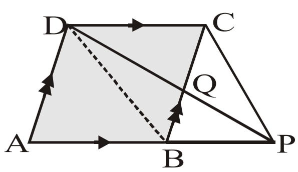
Q.28 In the given figure, the side AB of ||gm ABCD is produced to a point P. A line through A drawn parallel to CP meets CB produced in Q and the parallelogram PBQR is completed. Prove that: ar (|| gm ABCD) = ar (|| gm BPRQ).



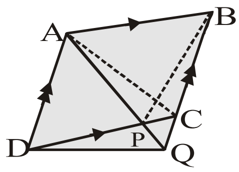
Q.29 In the adjoining figure, CE is drawn parallel to DB to meet AB produced at E. Prove that: ar (quad. ABCD) = ar (ΔDAE).



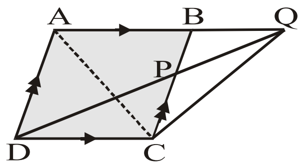
Q.30 In the adjoining figure, ABCD is a parallelogram. AB is produced to a point P and DP intersects BC at Q. Prove that: ar (ΔAPD) = ar (quad. BPCD).



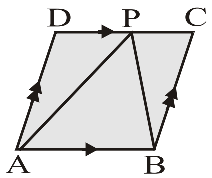
Q.31 In the adjoining figure, ABCD is a parallelogram. Any line through A cuts DC at a point P and BC produced at Q. Prove that: ar (ΔBPC) = ar (ΔDPQ).



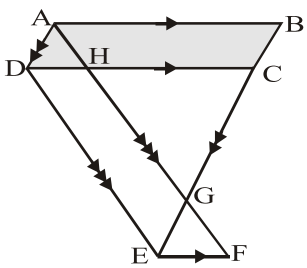
Q.32 In the adjoining figure, ABCD is a parallelogram. P is a point on BC such that BP : PC = 1 : 2. DP produced meets AB produced at Q. Given ar (ΔCPQ) = 20 cm^2. Calculate :
(i) ar (ΔCDP) (ii) ar (|| gm ABCD).



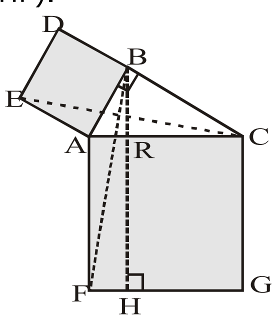
Q.33 In the adjoining figure, ABCD is a parallelogram. P is a point on DC such that ar (ΔAPD) = 25 cm^2 and ar (ΔBPC) = 15 cm^2. Calculate :
(i) ar (|| gm ABCD) (ii) DP : PC.



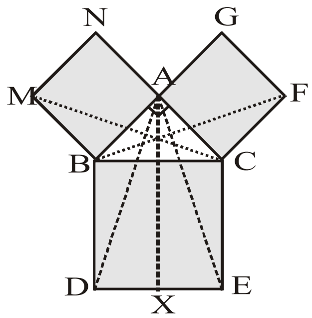
Q.34 In the given figure, AB || DC || EF, AD || BE and DE || AF. Prove that : ar (|| gm DEFH) = ar (|| gm ABCD).



Q.35 In the given figure, squares ABDE and AFGC are drawn on the side AB and hypotenuse AC of right triangle ABC and BH ⊥ FG. Prove that: (i) ΔEAC ≅ ΔBAF. (ii) ar (sq. ABDE) = ar (rect. ARHF).



Q.36 If fig, ABC is a right triangle right angled at A, BCED, ACFG and ABMN are squares on the sides BC, CA and AB respectively. Line segment AX ⊥ DE meets BC at Y. Show that



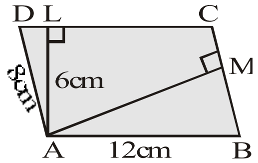
(i) ΔMBC ≅ ΔABD
(ii) ar (BYXD) = 2 ar (ΔMBC)
(iii) ar (BYXD) = ar (ABMN)
(iv) ΔFCB ≅ ΔACE
(v) ar (CYXE) = 2 ar (ΔFCB)
(vi) ar (CYXE) = ar (ACFG)
(vii) ar (BCED) = ar (ABMN) + ar.(ACFG)

ANSWER KEY

1.	48 cm^2
2.	12 cm
4.	(i) 90 cm^2, (ii) 45 cm^2, (iii) 45 cm^2
5.	(i) 128 cm^2, (ii) 128 cm^2
7.	(i) 80 cm^2, (ii) 12 cm, (iii) 16 cm, (iv) 100 cm^2, (v) 150 cm^2, 300 cm^2
8.	30 cm^2
9.	x = 13 cm, 40 cm^2
10.	40 cm^2
19.	(i) 2 : 1 , (ii) 1 : 6
32.	(i) 40 cm^2, (ii) 120 cm^2
33.	(i) 80 cm^2, (ii) 5 : 3

EXERCISE – III | MULTIPLE CHOICE QUESTIONS

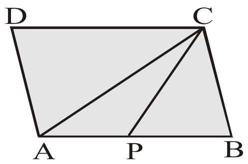
Q.1 In fig, ABCD is a parallelogram, AL $\perp$ CD and AM $\perp$ BC. If AB = 12 cm, AD = 8 cm and AL = 6 cm, then AM =



(A) 15 cm (B) 9 cm

(C) 10 cm (D) None of these

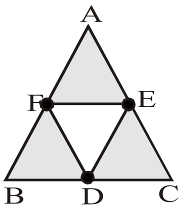
Q.2 In fig, ABCD is a parallelogram and P is mid-point of AB. If ar (APCD) = 36 cm^2, then ar (ΔABC) =



(A) 36 cm^2 (B) 48 cm^2

(C) 24 cm^2 (D) None of these

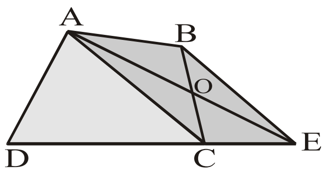
Q.3 In fig, if (ΔABC) = 28 cm^2, then ar (AEDF) =



(A) 21 cm^2 (B) 18 cm^2

(C) 16 cm^2 (D) None of these

Q.4 In fig, ABCD is a quadrilateral. BE $\parallel$ AC. BE meets DC (produced) at E. AE and BC intersect at O. Which one is the corect answer from the following?



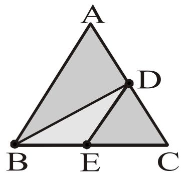
(A) ABEC is a parallelogram

(B) ar (ΔAOC) = ar (ΔBOE)

(C) ar (ΔOAB) = ar (ΔOCE)

(D) ar (ΔABE) = ar (ΔACE)

Q.5 In fig, D and E are the mid-points of the sides AC and BC respectiely of ΔABC. If ar (ΔBED) = 12 cm^2. then ar (ABED) =



(A) 36 cm^2 (B) 48 cm^2

(C) 24 cm^2 (D) None of these

Q.6 Two parallelograms stand on equal bases and between the same parallels. The ratio of their areas is

(A) 1 : 2 (B) 2 : 1

(C) 1 : 1 (D) 1 : 3

Q.7 If a rectangle and a parallelogram are equal in area and have the same base and are situated on the same side, then the quotient : $\frac{\text{Perimeter of rectangle}}{\text{Perimeter of } \parallel \text{gm}}$ is

(A) Equal to 1 (B) Greater than 1

(C) Less than 1 (D) Indeterminate

Q.8 If ABCD is a rectangle, E, F are the mid points of BC and AD respectively and G is any point on EF, then ΔGAB equals.

(A) $\frac{1}{2}$(ABCD) (B) $\frac{1}{3}$(ABCD)

(C) $\frac{1}{4}$(ABCD) (D) $\frac{1}{6}$(ABCD)

Q.9 D,E,F are mid points of the sides BC, CA & AB respectively of ΔABC, then area of $\parallel$ gm BDEF is equal to

(A) $\frac{1}{2}$ ar (ΔABC) (B) $\frac{1}{4}$ ar (ΔABC)

(C) $\frac{1}{3}$ ar (ΔABC) (D) $\frac{1}{6}$ ar (ΔABC)

Q.10 ABCD is a quadrilateral P,Q,R and S are the mid-points of AB, BC, CD and DA respectively, then PQRS is a

(A) Square (B) Parallelogram

(C) Trapezium (D) Kite

Q.11 Two parallelograms are on the same base and between the same paralles. The ratio of their areas is

(A) 2 : 1 (B) 1 : 2

(C) 1 : 1 (D) 3 : 1

Q.12 ABCD is a parallelogram and 'O' is the point of intersection of its diagonals $\overline{AC}$ and $\overline{BD}$. If the area of ΔAOD = 8 cm^2 the area of the parallelogram is

(A) 2 cm^2 (B) 4 cm^2

(C) 16 cm^2 (D) 32 cm^2

Q.13 A triangle and a rhombus are on the same base and between the same parallels. Then the ratio of the areas of the triangle and the rhombus is

(A) 1 : 1 (B) 1 : 2

(C) 1 : 3 (D) 1 : 4

Q.14 The area of a trepezium is 24 cm^2. The distance between its parallel sides is 4 cm If one of the parallel sides is 7 cm, the other parallel side is

(A) 5 cm (B) 8 cm

(C) 12 cm (D) 7 cm

Q.15 The area of a square is 16 cm^2. Its perimeter is

(A) 4 cm (B) 8 cm

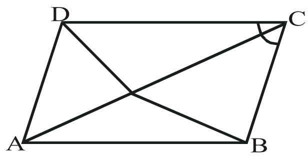
(C) 112 cm (D) 16 cm

Q.16 The ratio of the areas of two squares is 4 : 9. The ratio of their perimeters in the same order is

(A) 3 : 2 (B) 2 : 3

(C) 9 : 4 (D) 4 : 9

Q.17 In the given figure, P is a point in the interior of parallelogram ABCD. If the area of parallelogram ABCD is 60 cm^2, then area of ΔADP + area of ΔBPC =



(A) 15 cm^2 (B) 30 cm^2

(C) 45 cm^2 (D) 20 cm^2

Q.18 A parallelogram and a rectangle are on the same base and between the same parallel lines. Then the perimeter of the rectangle is

(A) Equal to the perimeter of the parallelogram

(B) Greater than the perimeter of the parallelogram

(C) Less than the perimeter of the parallelogram

(D) None of these

Q.19 The area of a rhombus is 220 cm^2. If one of its diagonals is 5 cm, the other diagonal is

(A) 4 cm (B) 8 cm

(C) 10 cm (D) 16 cm

Q.20 The diagonal of a square is 8 cm. Its area is

(A) 4 cm^2 (B) 16 cm^2

(C) 24 cm^2 (D) 32 cm^2

Q.21 If E, F,G and H are respectively the mid-points of the sides of a parallelogram ABCD, then ar(EFGH) is equal to

(A) ar(ABCD) (B) 2 × ar(ABCD)

(C) $\frac{1}{2}$ × ar(ABCD) (D) None of these

Q.22 In a ΔABC, E is the mid-point of median AD, then ar(ΔABC) is equal to

(A) 2 × ar(ΔBED) (B) 3 × ar(ΔBED)

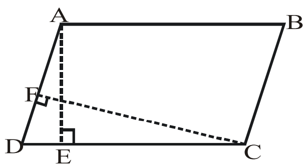
(C) 4 × ar(ΔBED) (D) None of these

Q.23 In a parallelogram ABCD, AB = 12 cm. The altitudes corresponding to the sides AB and AD are respectively 8 cm and 6 cm, then AD is equal to

(A) 6 cm (B) 12 cm

(C) 16 cm (D) 15 cm

Q.24 In figure, AD = 6 cm, CF = 10 cm and AE = 8 cm, then AB is



(A) 8 cm (B) 6.5 cm

(C) 7.5 cm (D) 9 cm

Q.25 If BD is one of the diagonals of a quadrilateral ABCD. AM and CN are the perpendiculars from A and C respectively on BD, then ar(ABCD) is equal to

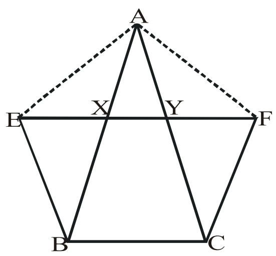
(A) BD × (AM + CN)

(B) $\frac{1}{2}$BD × (AM + CN)

(C) 2BD × (AM + CN)

(D) None of these

Q.26 In figure, XY is a line parallelogram to the side BC and ΔABC, BE || AC and CF || AB meet XY in E and F respectively. Also EX = FY, then ar(ΔABE) is equal to



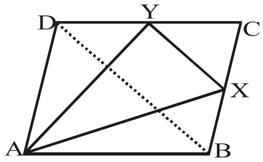
(A) ar(ΔABC)

(B) ar(ΔACF)

(C) ar(ΔXEB) + ar(ΔYFC)

(D) None of these

Q.27 ABCD is a parallelogram X and Y are the mid points of BC and CD respectively. Then, ar(parallelogram ABCD) is



(A) 4 × ar(ΔAXY)

(B) 2 × ar(ΔAXY)

(C) $\frac{8}{3}$ × ar(ΔAXY)

(D) None of these

Q.28 Two parallelograms are on the same base and between the same paralles. The ratio of their areas is

(A) 2 : 1 (B) 1 : 2

(C) 1 : 1 (D) 3 : 1

Q.29 ABCD is a parallelogram and 'O' is the point of intersection of its diagonals $\overline{AC}$ and $\overline{BD}$. If the area of ΔAOD = 8 cm^2 the area of the parallelogram is

(A) 2 cm^2 (B) 4 cm^2

(C) 16 cm^2 (D) 32 cm^2

Q.30 A triangle and a rhombus are on the same base and between the same parallels. Then the ratio of the areas of the triangle and the rhombus is

(A) 1 : 1 (B) 1 : 2

(C) 1 : 3 (D) 1 : 4

Q.31 The area of a trepezium is 24 cm^2. The distance between its parallel sides is 4 cm If one of the parallel sides is 7 cm, the other parallel side is

(A) 5 cm (B) 8 cm

(C) 12 cm (D) 7 cm

Q.32 The area of a square is 16 cm^2. Its perimeter is

(A) 4 cm (B) 8 cm

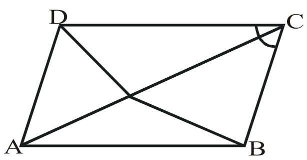
(C) 112 cm (D) 16 cm

Q.33 The ratio of the areas of two squares is 4 : 9. The ratio of their perimeters in the same order is

(A) 3 : 2 (B) 2 : 3

(C) 9 : 4 (D) 4 : 9

Q.34 In the given figure, P is a point in the interior of parallelogram ABCD. If the area of parallelogram ABCD is 60 cm^2, then area of ΔADP + area of ΔBPC =



(A) 15 cm^2 (B) 30 cm^2

(C) 45 cm^2 (D) 20 cm^2

Q.35 The area of a rhombus is 220 cm^2. If one of its diagonals is 5 cm, the other diagonal is

(A) 4 cm (B) 8 cm

(C) 10 cm (D) 16 cm

Q.36 A parallelogram and a rectangle are on the same base and between the same parallel lines. Then the perimeter of the rectangle is

(A) Equal to the perimeter of the parallelogram

(B) Greater than the perimeter of the parallelogram

(C) Less than the perimeter of the parallelogram

(D) None of these

Q.37 The diagonal of a square is 8 cm. Its area is

(A) 4 cm^2 (B) 16 cm^2

(C) 24 cm^2 (D) 32 cm^2

Q.38 If E, F,G and H are respectively the mid-points of the sides of a parallelogram ABCD, then ar(EFGH) is equal to

(A) ar(ABCD) (B) 2 × ar(ABCD)

(C) $\frac{1}{2}$ × ar(ABCD) (D) None of these

Q.39 In a ΔABC, E is the mid-point of median AD, then ar(ΔABC) is equal to

(A) 2 × ar(ΔBED) (B) 3 × ar(ΔBED)

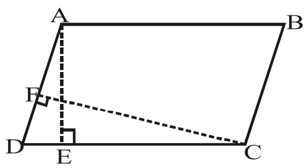
(C) 4 × ar(ΔBED) (D) None of these

Q.40 In a parallelogram ABCD, AB = 12 cm. The altitudes corresponding to the sides AB and AD are respectively 8 cm and 6 cm, then AD is equal to

(A) 6 cm (B) 12 cm

(C) 16 cm (D) 15 cm

Q.41 In figure, AD = 6 cm, CF = 10 cm and AE = 8 cm, then AB is



(A) 8 cm (B) 6.5 cm

(C) 7.5 cm (D) 9 cm

Q.42 If BD is one of the diagonals of a quadrilateral ABCD. AM and CN are the perpendiculars from A and C respectively on BD, then ar(ABCD) is equal to

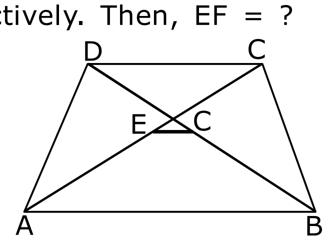
(A) BD × (AM + CN)

(B) $\frac{1}{2}$BD × (AM + CN)

(C) 2BD × (AM + CN)

(D) None of these

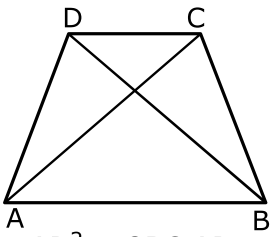
Q.43 In a trapezium ABCD, if E and F be the mid-points of the diagonals AC and BD respectively. Then, EF = ?



(A) $\frac{1}{2}$AB (B) $\frac{1}{2}$CD

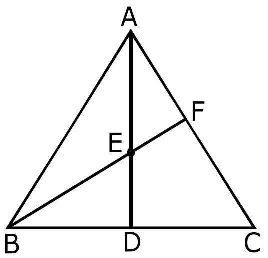
(C) $\frac{1}{2}$(AB + CD) (D) $\frac{1}{2}$(AB − CD)

Q.44 In a trapezium ABCD, if AB||CD, then (AC^2 + BD^2) = ?



(A) $BC^2 + AD^2 + 2BC.AD$
(B) $AB^2 + CD^2 + 2AB.CD$
(C) $AB^2 + CD^2 + 2AD.BC$
(D) $BC^2 + AD^2 + 2AB.CD$

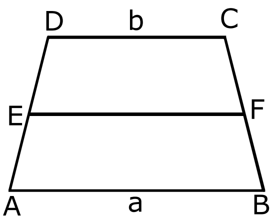
Q.45 In the given figure, AD is a median of ΔABC and E is the mid-point of AD. If BE is joined and produced to meet AC in F, then AF =?



(A) $\frac{1}{2}$AC (B) $\frac{1}{3}$AC

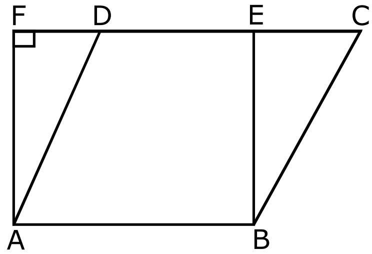
(C) $\frac{2}{3}$AC (D) $\frac{3}{4}$AC

Q.46 In the given figure ABCD is a trapezium in which AB||DC such that AB = a cm and DC = b cm. If E and F are the midpoints of AD and BC respectively. Then, ar(ABFE) : ar(EFCD) = ?



(A) a : b
(B) (a + 3b) : (3a + b)
(C) (3a + b) : (a + 3b)
(D) (2a + b) : (3a + b)

Q.47 In the given figure, a ||gm ABCD and a rectangle ABEF are of equal area. Then,



(A) perimeter of ABCD = perimeter of ABEF
(B) perimeter of ABCD < perimeter of ABEF
(C) perimeter of ABCD > perimeter of ABEF
(D) perimeter of ABCD = $\frac{1}{2}$(perimeter of ABEF)

ANSWER KEY

1.	B	**2.**	C	**3.**	D	**4.**	C
5.	A	**6.**	C	**7.**	C	**8.**	C
9.	A	**10.**	B	**11.**	C	**12.**	D
13.	B	**14.**	A	**15.**	D	**16.**	B
17.	B	**18.**	C	**19.**	B	**20.**	D
21.	C	**22.**	C	**23.**	C	**24.**	C
25.	B	**26.**	B	**27.**	C	**28.**	C
29.	D	**30.**	B	**31.**	A	**32.**	D
33.	B	**34.**	B	**35.**	B	**36.**	C
37.	D	**38.**	C	**39.**	C	**40.**	C
41.	C	**42.**	B	**43.**	D	**44.**	D
45.	B	**46.**	C	**47.**	C		

EXERCISE – IV | OLYMPIAD QUESTIONS

MULTIPLE CHOICE TYPE QUESTIONS

1. Ratio of lateral surface areas of two cylinders with equal heights is :

(A) H : h (B) R : r (C) 1 : 2 (D) None of these

2. Ratio of volumes of two cylinders with equal radii are :

(A) R : r (B) H : h (C) $R^2 : r^2$ (D) None of these

3. The lateral surface area of cylinder is 176 cm^2 and base area 38.5 cm^2. The its volume is _____.

(A) 830 cm3 (B) 380 cn^3 (C) 308 cm^3 (D) 803 cm^3

4. A cylinderical vessel contains 49.896 litres of liquid. cost of painting its CSA at 2 paise/sq cm is Rs. 95.04. Then its total surface area is _______.

(A) 5724 cm^2 (B) 7524 cm^2 (C) 5742 cm^2 (D) None of these

5. The area of the base of a cone is 616 sq cm. Its height is 48 cm. Then its total surface area is _____.

(A) 2816 cm^2 (B) 2861 cm^2 (C) 2618 cm^2 (D) 2681 cm^2

6. A vessel is in conical shape. If its volume is 33.264 litres and height is 72 cm, the cost of repairing its CSA at Rs. 12/sq m is :

(A) 5.94 (B) 6.94 (C) 7.95 (D) None of these

7. From a circle of radius 15 cm a sector with 216° angle is cut out and its bounding radii are bent so as to form a cone. Then its volume is :

(A) 1081.3 cm^3 (B) 1071.3 cm^3 (C) 1018.3 cm^3 (D) None of these

8. A hemispherical bowl is made of steel of 0.25 cm thickness. The inner radius of the bowl is 5 cm. The volume of steel used is ______.

(A) 42.15 cm^3 (B) 41.52 cm^3 (C) 41.25 cm^3 (D) None of these

9. A cuboidal metal of dimensions 44 cm × 30 cm × 15 cm was melted and case into a cylinder of height 28 cm. Its radius is ______.

(A) 20 cm (B) 15 cm (C) 10 cm (D) None of these

10. A piece of metal pipe is 77 cm long with inside diameter of the cross section as 4 cm. If teh outer diameter is 4.5 cm and the metal weighs 8 gm/cu cm, the weight of pipe is ______.

(A) 2.057 kg (B) 20.57 kg (C) 205.7 kg (D) None of these

11. A circus tent is in the form of a cone over a cylinder. The diameter of the base is 9 m, the height of cylindrical part is 4.8 m and the total height of the tent is 10.8 m. The canvas required for the tent is ______.

(A) 24.184 sq m (B) 2418.4 sq m (C) 241.84 sq m (D) None of these

12. The diameter of a copper sphere is 6 cm. It is beaten and drawn into a wire of diameter 0.2 cm. The length of wire is ______.

(A) 36 cm (B) 360 m (C) 3600 cm (D) None of these

13. A hollow sphere of internal and external diameters 4 cm and 8 cm respectively is melted into a cone of base diameter 8 cm. Find the height of the cone.

(A) 14 cm (B) 12 cm (C) 16 cm (D) None of these

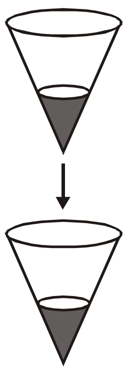
14. The radius of the cylinder whose lateral surface area is 704 cm^2 and height 8 cm is :

(A) 6 cm (B) 4 cm (C) 8 cm (D) 14 cm

15. The ratio of the volume and surface area of a sphere of unit radius :

(A) 4 : 3 (B) 3 : 4 (C) 1 : 3 (D) 3 : 1

16. Two identical right circular cones each of height 2 cm are placed as showin in diagram (each is vertical, apex downward). At the start, the upper cone is full of water and lower cone is empty. Then water drips down through a hole in the apex of upper cone into the lower cone. The height of water in the lower cone at the moment when height of water in uper cone is 1 cm is :



(A) 1 cm (B) $\sqrt{\frac{1}{2}}$ cm

(C) $\sqrt[3]{\frac{1}{4}}$ cm (D) $\sqrt[3]{7}$ cm

17. A sphere and a cube are of the same height. The ratio of their volume is :

(A) 3 : 4 (B) 21 : 11

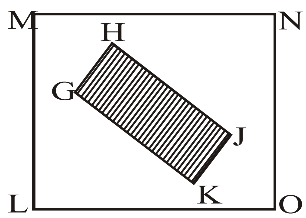
(C) 4 : 3 (D) 11 : 21

18. The largest sphere is cut off from a cube of side 5 cm. The volume of the sphere will be :

(A) 27π cm^3 (B) 30π cm^3

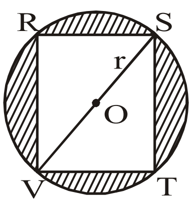
(C) 108π cm^3 (D) $\frac{125\pi}{6}cm^3$

19. In the figure below, LMNO and GHJK are rectangles where GH = $\frac{1}{2}$LM and HJ = $\frac{1}{2}$MN. What fraction of the region is bounded by LMNO that is not shaded?



(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$

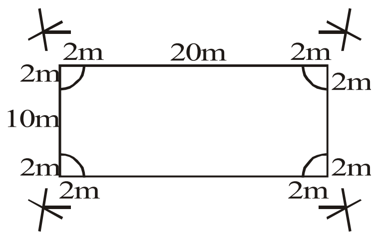
20. In the figure below, RSTV is a square inscribed in a circle with centre O and radius r. The total area of shaded region is :



(A) $r^2(\pi - 2)$ (B) $2r(2 - \pi)$

(C) $\pi(r^2 - 2)$ (D) $\pi r^2 - 8r$

21. Correct the perimeter of the figure given below to one decimal place :



(A) 56.0 m (B) 56.6 m

(C) 57.7 m (D) 57.9 m

22. A hollow spherical ball whose inner radius is 4 cm is full of water. Half of the water is transferred to a conical cup and it completely filled' he cup. If the height of the cup is 2 cm, then the radius of the base of cone, in cm is :

(A) 4 (B) 8 π (C) 8 (D) 16

23. The largest volume of a cube that can be enclosed in a sphere of diameter 2 cm is (in cm^3).

(A) 1 (B) $2\sqrt{2}$

(C) π (D) $\frac{8}{3\sqrt{3}}$

24. Each side of a cube is increased by 50%. Then the surface area of the cube increases by :

(A) 50% (B) 100%

(C) 125% (D) 150%

25. The number of surfaces in right circular cylinder is :

(A) 1 (B) 2 (C) 3 (D) 4

26. The edge of a cube is 20 cm. How many small cubes of 5 cm edge can be formed from this cube?

(A) 4 (B) 32

(C) 64 (D) 100

ANSWER KEY

1.	B	**2**	B	**3.**	C	**4.**	B
5.	A	**6.**	A	**7.**	C	**8.**	C
9.	B	**10.**	A	**11.**	C	**12.**	C
13.	A	**14.**	D	**15.**	C	**16.**	D
17.	D	**18.**	D	**19.**	D	**20.**	A
21.	B	**22.**	C	**23.**	B	**24.**	C
25.	C	**26.**	C				